H

National Qualifications
CFE Higher Mathematics - Specimen Paper A

Mathematics
Paper 1
(Non-Calculator)

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a}\cdot\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a}\cdot\boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Attempt ALL questions

Total marks — 60

1. Given that $g(x) = x^{-2}(x^3 - \frac{1}{x^2})$, find $g'(x)$, giving your answer with positive indices. **4**

2. Find the points of intersection of the curve with equation $y = 6x^2 - x^3$ and the line with equation $y = 3x + 10$. **5**

3. Given that $\boldsymbol{a} = 3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$, find the value of cos θ, in its simplest form, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. **5**

4. Find the range of values for k, given that the equation $2x^2 + 8 = kx$ has **no real roots** and $k \in R$. **4**

5. **(a)** Find the gradient of a line which is perpendicular to $y - x = -5$. **2**

(b) What angle does this perpendicular line makes with the positive direction of the x – axis? **2**

6. **(a)** Find an equivalent expression for $\cos(x + 30)°$. **1**

(b) Hence, or otherwise, determine the exact value of cos 75°. **3**

7. **(a)** Show that $(x - 3)$ is a factor of $2x^3 - 3x^2 - 11x + 6$ **3**

(b) Hence, factorise $2x^3 - 3x^2 - 11x + 6$ fully. **2**

8. Two functions, defined on suitable domains, are given as $f(x) = 2x^2 - 1$ and $g(x) = 2 - x$.

(a) Find an expression, in its simplest form, for $f(g(x))$. **3**

(b) By expressing $f(g(x))$ in the form $a(x+p)^2 + q$, state the turning point of the graph of $y = f(g(x))$. **4**

(c) Sketch the graph of $y = f(g(x)) - 2$ showing clearly its turning point and intercept with the y – axis. **2**

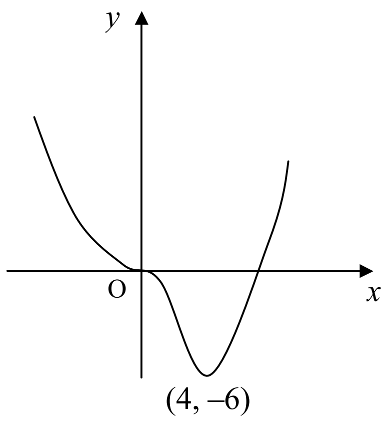
9. A$(1,\ 2)$, B$(7,\ 6)$ and C$(-7, 14)$ are the vertices of a triangle.

(a) By using gradients, show clearly that triangle ABC is right-angled at A. **3**

(b) M is the mid-point of AC. Find the equation of the median BM. **3**

10. Find the rate of change of the function $y = 3\cos^3 x$ when $x = \frac{2\pi}{3}$. **4**

11.

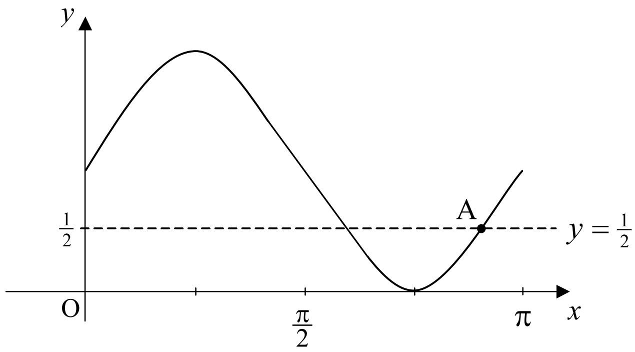


The diagram shows part of the graph of $y = f(x)$.

It has stationary points at (0, 0) and $(4, -6)$.

Make a sketch of $y = f'(x)$. **2**

12. The diagram below shows part of the graph of $y = \sin 2x + 1$, for $0 \le x \le \pi$, and the line with equation $y = \frac{1}{2}$.



Find the coordinates of the point A. **4**

13. For what value(s) of x is the function

$3 + 75x - x^3$ decreasing ? **4**

[END OF QUESTION PAPER]

H

National Qualifications
CFE Higher Mathematics - Specimen Paper A

Mathematics
Paper 2
(Calculator)

Duration — 1 hour and 30 minutes

Total marks — 70

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

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or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

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Table of standard integrals:

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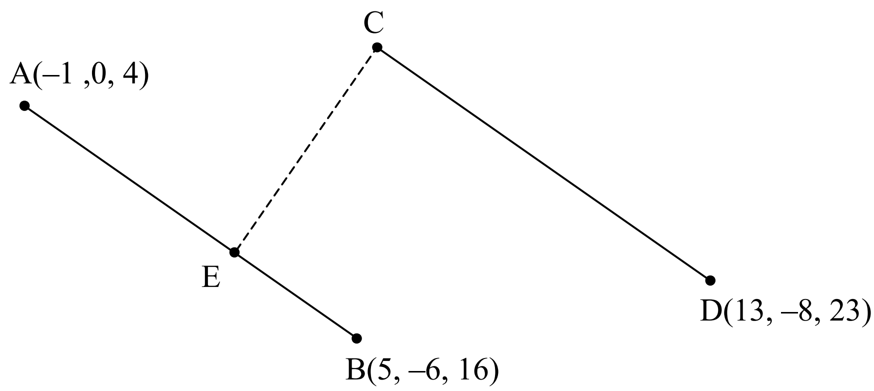
$\sin ax$ $\cos ax$	$-\frac{1}{a}\cos ax + C$ $\frac{1}{a}\sin ax + C$

Attempt ALL questions

Total marks — 70

1. Three points have coordinates A(–1, 0, 4), B(5, –6, 16) and D(13, –8, 23).



(a) Establish the coordinates of C if AB is parallel and equal in length to CD. **1**

(b) The point E divides the line AB in the ratio $2:1$.
Find the coordinates of E. **4**

(c) Hence prove that CE is perpendicular to AB. **3**

2. Two unique sequences are defined by the following recurrence relations

$U_{n+1} = pU_n + 6$ and $U_{n+1} = p^2U_n + 9$, where p is a constant.

(a) If both sequences have the same limit, find the value of p. **4**

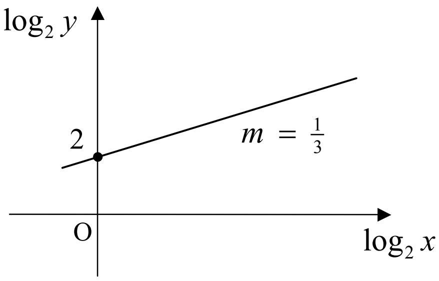
(b) For both sequences $U_0 = 100$, find the difference between their first terms. **3**

3. A function is defined as: $f(x) = x^3 - 2x^2 + x - 1$.

(a) Find the stationary values of the function and the corresponding values of x.
Determine the nature of each stationary point. **7**

(b) Establish the equation of the tangent to the curve $y = f(x)$ at the point where $x = 2$. **3**

4. The diagram, which is not drawn to scale, shows part of a graph of $\log_2 x$ against $\log_2 y$.
The straight line has a gradient of $\frac{1}{3}$ and passes through the point (0, 2).

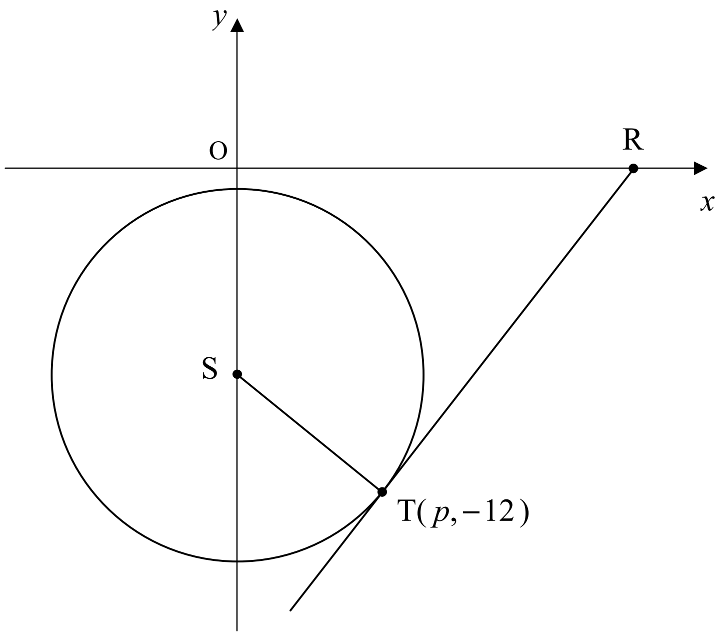


Find an equation connecting x and y. **4**

5. Given that $3\log_y(2x) = \log_y(x) + 1$ find a relationship connecting x and y. **4**

6. The circle, centre S, has as its equation $x^2 + y^2 + 16y + 12 = 0$.

T$(p, -12)$ is a point of tangency.



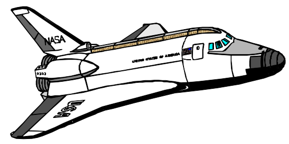
(a) Find the value of p, the x-coordinate of T. **2**

(b) Write down the coordinates of S, the centre of the circle. **1**

(c) Find the equation of the tangent through T and hence state the coordinates of R. **4**

(d) Establish the equation of the circle which passes through the points S, T and R. **3**

7. On a third stage burn the space shuttle uses fuel from a secondary tank.



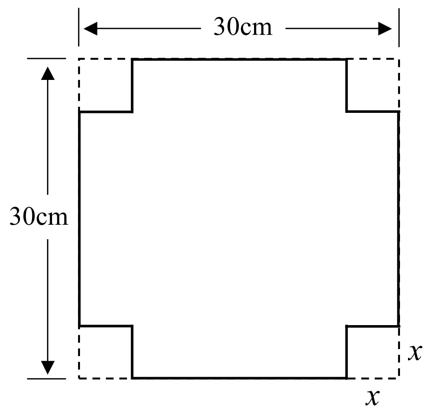
The mass of fuel left in the tank after a burn of t seconds can be calculated using the formula

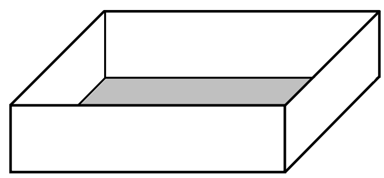
$M_t = M_0 e^{-0{\cdot}03t}$, where M_0 is the initial payload in tonnes and M_t is the fuel remaining after t seconds.

Calculate, to the nearest second, how long the shuttle takes to use up 60% of the fuel in its tank. **5**

8. From a square sheet of metal of side 30 centimetres, equal squares of side x centimetres are removed from each corner.

The sides are then folded up and sealed to form an open cuboid.





(a) Show that the volume of this resulting cuboid is given by

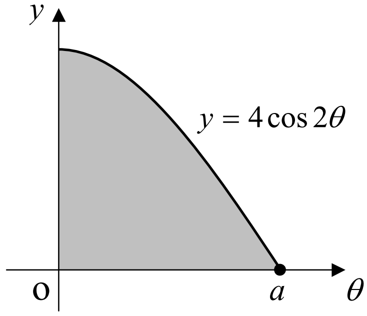
$$V(x) = 4x^3 - 120x^2 + 900x.$$ **3**

(b) If the cuboid is to have **maximum** possible volume, what size of square should be removed from each corner? **5**

(c) How many litres of water would this particular cuboid hold? **1**

9. A funtion is defined as $f(\theta) = 4\cos 2\theta$ where $0 \le \theta \le \pi$.

Part of the graph of $y = f(\theta)$ is shown.



(a) State the value of a in radians. **2**

(b) Calculate the shaded area in square units. **4**

10. **(a)** Express $3\cos x^\circ + \sqrt{3}\sin x^\circ + 3$ in the form $k\cos(x - \alpha)^\circ + 3$ where $0 < \alpha < 360$ and $k > 0$. **3**

(b) Hence solve the equation $f(x) = 0$ for $200 < x < 360$. **4**

[**END OF QUESTION PAPER**]

CFE Higher Specimen Paper A – Marking Scheme Paper 1

<table>
<tr><th></th><th>Give 1 mark for each •</th><th>Illustration(s) for awarding each mark</th></tr>
<tr><td>1</td><td>ans: $1+\frac{4}{x^5}$ (4 marks)

•1 brings power up
•2 prepares to differentiate
•3 differentiates
•4 write with positive indices</td><td>

•1 $x^{-2}(x^3 - x^{-2})$
•2 $x - x^{-4}$
•3 $1 + 4x^{-5}$
•4 $1+\frac{4}{x^5}$
Note: mark 3 can only be awarded when differentiating a negative power.</td></tr>
<tr><td>2</td><td>ans: (–1, 7); (2, 16); (5, 25) (5 marks)

•1 equates equations
•2 sets to zero
•3 factorises fully
•4 calculates x - coordinates
•5 calculates y - coordinates</td><td>

•1 $3x+10 = 6x^2 - x^3$;
•2 $x^3 - 6x^2 + 3x + 10 = 0$
•3 $(x+1)(x-2)(x-5) = 0$
•4 $x = -1;\ 2;\ 5$
•5 (–1, 7); (2, 16); (5, 25)</td></tr>
<tr><td>3</td><td>ans: $\frac{1}{2}$ (5 marks)

•1 states components of both vectors
•2 finds scalar product
•3 finds magnitude of both vectors
•4 substitutes into formula
•5 simplifies</td><td>

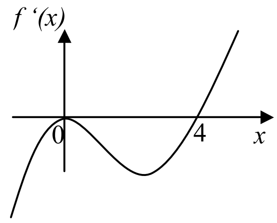
•1 $\boldsymbol{a} = \begin{pmatrix}3\\2\\1\end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix}2\\-1\\3\end{pmatrix}$
•2 $\boldsymbol{a}\,.\,\boldsymbol{b} = 6 - 2 + 3 = 7$
•3 $|\boldsymbol{a}| = \sqrt{14}$ $|\boldsymbol{b}| = \sqrt{14}$
•4 $\cos\theta = \frac{7}{\sqrt{14}\sqrt{14}}$
•5 $\cos\theta = \frac{1}{2}$</td></tr>
<tr><td>4</td><td>ans: $-8 < x < 8$ (4 marks)

•1 knows condition for non real roots
•2 simplifies
•3 factorises
•4 correct range</td><td>

•1 $b^2 - 4ac < 0$ for non real roots – stated or implied at•2
•2 $b^2 - 4ac = k^2 - 4.2.8 < 0$
•3 $(p+8)(p-8) < 0$
•4 $-8 < x < 8$</td></tr>
</table>

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	**ans: –1** **(2 marks)**	
	•[1] rearranges equation to find gradient •[2] states perpendicular gradient	•[1] $y = x - 5$ •[2] -1
(b)	**ans: 135° or $\frac{3\pi}{4}$** **(2 marks)**	
	•[1] using $\tan\theta = m$ •[2] calculates angle	•[1] $\tan\theta = -1$ •[2] $\theta = 135°$ or $\theta = \frac{3\pi}{4}$
6(a)	**ans: $\cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$** **(1 mark)**	
	•[1] correct expansion	•[1] $\cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$
(b)	**ans: $\frac{\sqrt{3}-1}{2\sqrt{2}}$** **(3 marks)**	
	•[1] any expression equivalent to $\cos 75^\circ$ •[2] correct exact values substituted •[3] establishes d	•[1] $\cos(45 + 30)^\circ$ or equivalent •[2] $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ •[3] $\frac{\sqrt{3}-1}{2\sqrt{2}}$
7(a)	**ans: –1** **(3 marks)**	
	•[1] knows to use $x = 3$	•[1] $\begin{array}{c\|l} 3 & \\ \hline \end{array}$
	•[2] completes synthetic division	•[2] $\begin{array}{c\|cccc} 3 & 2 & -3 & -11 & 6 \\ & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$
	•[3] recognition of zero remainder	•[3] $(x - 3)$ is a factor as remainder is zero
(b)	**ans: $(x - 3)(2x - 1)(x + 2)$** **(2 marks)**	
	•[1] identify quotient •[2] factorises fully	•[1] $(x-3)(2x^2+3x-2)$ •[2] $(x-3)(2x-1)(x+2)$
	Alternative methods of showing $(x - 3)$ is a factor, such as long division, inspection and evaluating are acceptable.	

	Give 1 mark for each •	Illustration(s) for awarding each mark
8(a)	**ans: $2x^2 - 8x + 7$ (3 marks)**	
	•1 correct substitution •2 expanding brackets •3 simplifying	•1 $f(g(x)) = 2(2-x)^2 - 1$ •2 $2(4-4x+x^2)-1$ •3 $2x^2 - 8x + 7$
(b)	**ans: $2(x-2)^2 - 1$; $(2, -1)$ (4 marks)**	
	•1 identify common factor •2 complete the square •3 process for q •4 state turning point	•1 $(x^2 - 4x)$ stated or implied at •2 •2 $2(x-2)^2$…. •3 $2(x-2)^2 - 1$ •4 $(2, -1)$
(c)	**ans: graph drawn (2 marks)**	
	•1 correct shape •2 annotation, including y-axis intercept	•1 parabola with minimum T.P. •2 T.P. at ((2, –3) and y – intercept at (0, 5)
9(a)	**ans: proof (3 marks)**	
	•1 finds gradient of AB •2 finds gradient of AC •3 recognises why right angled at A	•1 $m_{AB} = \frac{2}{3}$ •2 $m_{AC} = -\frac{3}{2}$ •3 since $m_{AB} \times m_{AC} = -1$ triangle is right angled at A
(b)	**ans: $5y + x = 37$ (3 marks)**	
	•1 finds midpoint of AC •2 finds gradient of BM •3 substitutes into equation of straight line	•1 midpoint AC = (–3, 8) •2 $m_{BM} = -\frac{1}{5}$ •3 $y - 8 = -\frac{1}{2}(x+3)$
10	**ans: $-\frac{9\sqrt{3}}{8}$ (4 marks)**	
	•1 starts to differentiate •2 completes differentiation •3 substitutes •4 evaluates	•1 $9\cos^2 x$………. •2 $\ldots\ldots \times -\sin x$ •3 $-9\cos^2\frac{2\pi}{3} \times \sin\frac{2\pi}{3}$ •4 $-9 \times (-\frac{1}{2})^2 \times \frac{\sqrt{3}}{2} = -\frac{9\sqrt{3}}{8}$

	Give 1 mark for each	Illustration(s) for awarding each mark
11	**ans: graph drawn** **(2 marks)** ●[1] correct shape ●[2] correct intercepts with x - axis	●[1] ●[2] 
12	**ans:** $\mathbf{A\left(\frac{11\pi}{12}, \frac{1}{2}\right)}$ **(4 marks)** ●[1] equates line & curve, reorganises ●[2] finds values for $2x$ ●[3] finds values for x ●[4] states coordinates of A	●[1] $\sin 2x + 1 = \frac{1}{2}; \sin 2x = -\frac{1}{2}$ ●[2] $2x = \frac{7\pi}{6}, \frac{11\pi}{6}$ ●[3] $x = \frac{7\pi}{12}, \frac{11\pi}{12}$ ●[4] $A\ \left(\frac{11\pi}{12}, \frac{1}{2}\right)$
13	**ans:** $\mathbf{x < -5}$ **or** $\mathbf{x > 5}$ **(4 marks)** ●[1] knows to differentiate ●[2] knows derivative < 0 ●[3] attempts to solve for x ●[4] answer	●[1] $75 - 3x^2$ ●[2] $75 - 3x^2 < 0$ ●[3] graph drawn (or any acceptable method) ●[4] $x < -5$ or $x > 5$
		Total: 60 marks

CFE Higher Specimen Paper A – Marking Scheme Paper 2

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans:** C(7,-2,11) **(1 mark)**	
	•1 states coordinates of C	•1 stepping out from B to A is the same from D to C <u>or</u> by vector algebra
(b)	**ans:** E(3,-4,12) **(4 marks)**	
	•1 for using ratio	•1 $\overrightarrow{AE} = 2\overrightarrow{EB}$ (or equivalent)
	•2 introducing vector algebra	•2 $e - a = 2(b - e)$
	•3 introducing correct column vectors	•3 &•4 $3e = 2b + a \Rightarrow 3e = \begin{pmatrix} 9 \\ -12 \\ 36 \end{pmatrix} \therefore e = \begin{pmatrix} 3 \\ -4 \\ 12 \end{pmatrix}$
	•4 working to answer	
(c)	**ans:** proof **(3 marks)**	
	•1 for scalar product = 0	•1 If perp. then $\overrightarrow{EC}.\overrightarrow{AB} = 0$ (or equiv.)
	•2 for two correct column vectors	•2 $\overrightarrow{EC} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix}$
	•3 calculating scalar product thus proof	•3 $\overrightarrow{EC}.\overrightarrow{AB} = 4(6) + 2(-6) + (-1)(12) = 0$
2(a)	**ans:** $p = 0{\cdot}5$ **(4 marks)**	
	•1 gives expression for both limits	•1 $L = \frac{6}{1-p}$; $L = \frac{9}{1-p^2}$
	•2 equates limits	•2 $\frac{6}{1-p} = \frac{9}{1-p^2}$
	•3 starts to solve	•3 $6 - 6p^2 = 9 - 9p$; $6p^2 - 9p + 3 = 0$
	•4 solves and discards	•4 $3(2p-1)(p-1) = 0$; $p = 0\cdot5$ or $p = 1$
(b)	**ans:** **22** **(3 marks)**	
	•1 finds 1st term for one RR	•1 $U_1 = \frac{1}{2}(100) + 6 = 56$
	•2 finds 1st term for other RR	•2 $U_1 = (\frac{1}{2})^2(100) + 6 = 34$
	•3 calculates difference in terms	•3 56 – 34 = 22

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	**ans:** $-\frac{23}{27}, x=\frac{1}{3}$, **max :** $-1, x=1$, **min** **(7 marks)**	
	•[1] knowing to differentiate and equate to 0	•[1] $f'(x)=3x^2-4x+1=0$
	•[2] making $f'(x)=0$	•[2] ……..0
	•[3] solving	•[3] $x=\frac{1}{3}$ or 1
	•[4] substituting into $f(x)$	•[4] $(\frac{1}{3})^3-2(\frac{1}{3})^2+\frac{1}{3}-1$ or $(1)^3-2(1)^2+1-1$
	•[5] evaluating	•[5] $-\frac{23}{27}, -1$
	•[6] setting up table of values or second derivative	•[6] $\begin{array}{c\|ccccc} & \to & \frac{1}{3} & \to & 1 & \to \\ \hline f'(x) & + & 0 & - & 0 & + \\ \text{shape} & \nearrow & \to & \searrow & \to & \nearrow \end{array}$
	•[7] identifies correct nature of each value	•[7] max at $(\frac{1}{3}, -\frac{23}{27})$; min at (1, -1)
(b)	**ans:** $y=5x-9$ **(3marks)**	
	•[1] knows how to find gradient	•[1] $f'(2)=3(2)^2-4(2)+1=5=m$
	•[2] finds point on tangent	•[2] $f(2)=(2)^3-2(2)^2+2-1=1$
	•[3] subs into $(y-b)=m(x-a)$ & arrange	•[3] $(y-1)=5(x-2)$
4.	**ans:** $y=4x^{\frac{1}{3}}$ **(4 marks)**	
	•[1] knowing original form $y=kx^n$ (stated or implied)	•[1] original data in form $y=kx^n$
	•[2] gradient is power	•[2] power = gradient $\therefore\ n=\frac{1}{3}$
	•[3] finding k	•[3] y-intercept, $\log_2 k=2$, $2^2=k=4$
	•[4] final equation	•[4] $y=4x^{\frac{1}{3}}$
5	**ans:** $y=8x^2$ **(4 marks)**	
	•[1] for bringing up power of 3	•[1] $\log_y (2x)^3 = $ …….. (or equivalent)
	•[2] for taking logs to the same side	•[2] $\log_y (2x)^3-\log_y (x)=1$
	•[3] for knowing to divide to combine	•[3] $\log_y (8x^2)=1$
	•[4] changing to index form for answer	•[4] $y^1=8x^2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
6(a)	**ans:** $p = 6$ **(2 marks)**	
	•[1] subs into equation of circle •[2] solves for p	•[1] $p^2 + 144 - 192 + 12 = 0$ •[2] $p^2 = 36; p = 6$
(b)	**ans:** **(0, – 8)** **(1 mark)**	
	•[1] states centre of circle	•[1] (0, – 8)
(c)	**ans:** $2y = 3x - 42$; **R(14, 0)** **(4 marks)**	
	•[1] finds gradient of ST •[2] finds gradient of tangent •[3] subs into equation of straight line •[4] finds coords of point R	•[1] $m_{ST} = -\frac{2}{3}$ •[2] $m_{tan} = \frac{3}{2}$ •[3] $y + 12 = \frac{3}{2}(x - 6)$ [or equivalent] •[4] $3x - 42 = 0;\ x = 14\ \ (14, 0)$
(d)	**ans:** $(x - 7)^2 + (y + 4)^2 = 65$ **(3 marks)**	
	•[1] finds midpoint of SR (centre of circle) •[2] finds radius •[3] subs into equation of circle	•[1] centre of circle (7, – 4) •[2] $\sqrt{65}$ •[3] $(x - 7)^2 + (y + 4)^2 = 65$
7	**ans:** 31 seconds **(5 marks)**	
	•[1] for solving exponential to $0 \cdot 4$ •[2] taking logs of both sides •[3] releasing the power •[4] knowing $\log_e e = 1$ •[5] making t the subject + calc. answer	•[1] $e^{-0 \cdot 03t} = 0 \cdot 4$ (or equivalent) •[2] $\log_e e^{-0 \cdot 03t} = \log_e 0 \cdot 4$ •[3] $-0 \cdot 03t \log_e e = \log_e 0 \cdot 4$ •[4] $-0 \cdot 03t = \log_e 0 \cdot 4$ •[5] $t = \dfrac{\log_e 0 \cdot 4}{-0 \cdot 03} = 30 \cdot 54$ seconds
8(a)	**ans:** **proof** **(3 marks)**	
	•[1] gives expression for length and breadth •[2] subs into formula and starts to simplify •[3] completes simplification to answer	•[1] $(30 - 2x)$ •[2] $x(30 - 2x)^2$ •[3] $x(900 - 120x + 4x^2)$
(b)	**ans:** $x = 5$ **(5 marks)**	
	•[1] knows to make derivative = 0 •[2] takes derivative •[3] factorises •[4] solves and discards •[5] justifies answer	•[1] $V'(x) = 0$ •[2] $12x^2 - 240x + 900 = 0$ •[3] $12(x - 5)(x - 15) = 0$ •[4] $x = 5$ •[5] nature table or 2nd derivative
(c)	**ans:** **2 litres** **(1 mark)**	
	•[1] calculates volume	•[1] $20 \times 20 \times 5 = 2000\text{cm}^3 = 2$ litres

	Give 1 mark for each •	Illustration(s) for awarding each mark
9(a)	**ans:** $a = \frac{\pi}{4}$ **(2 marks)**	
	•[1] solving $\cos 2\theta$ to zero •[2] answer	•[1] $\cos 2\theta = 0$ •[2] $2\theta = \frac{\pi}{2}$, $\theta = \frac{\pi}{4}$
(b)	**ans:** 2 units^2 **(4 marks)**	
	•[1] setting up integral •[2] integrating •[3] substituting limits •[4] answer	•[1] $A = \int_0^{\frac{\pi}{4}} 4\cos 2\theta \; d\theta$ •[2] $\ldots \; 4 \times \frac{1}{2}\sin 2\theta \quad \therefore [2\sin 2\theta]_0^{\frac{\pi}{4}}$ •[3] $A = (2\sin 2(\frac{\pi}{4})) - (2\sin 0)$ •[4] $A =$ 2 units^2
10(a)	**ans:** $\mathbf{\sqrt{12}\cos(x-30)^o + 3}$ **(3 marks)**	
	•[1] finds k •[2] finds tan α •[3] finds α	•[1] $k = \sqrt{9+3} = \sqrt{12}$ •[2] $\tan\alpha = \frac{\sqrt{3}}{3}$ •[3] $\alpha = 30°$
(b)	**ans:** $\mathbf{240^o}$ **(4 marks)**	
	•[1] equates to 0 •[2] simplifies •[3] finds values •[4] discards	•[1] $\sqrt{12}\cos(x-30)^o + 3 = 0$ •[2] $\cos(x-30)^o = -\frac{3}{\sqrt{12}}$ •[3] $x = 180^o; 240^o$ •[4] 240^o
		Total: 70 marks