H

National Qualifications
CFE Higher Mathematics - Specimen Paper B

Mathematics
Paper 1
(Non-Calculator)

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Attempt ALL questions

Total marks — 60

1. A sequence is defined by the recurrence relation $U_{n+1} = 0 \cdot 6U_n + 8$.

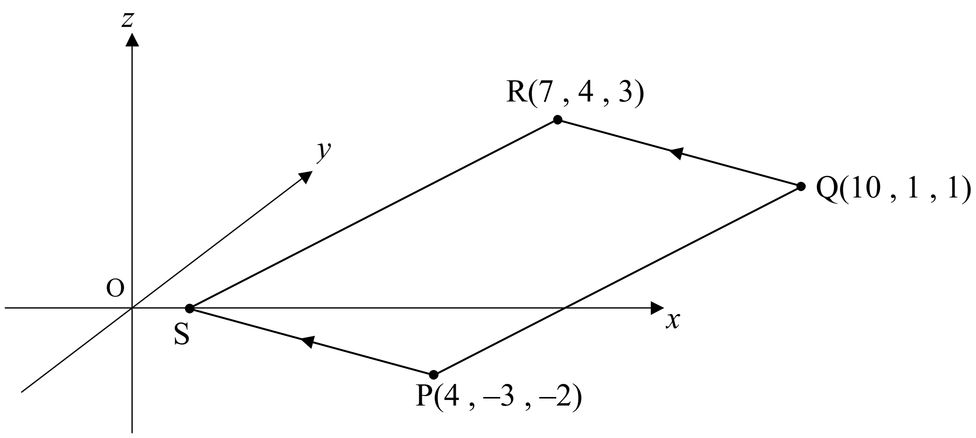
(a) Explain why this sequence has a limit as $n \to \infty$. **1**

(b) Find the limit of this sequence. **2**

(c) Given that $L - U_1 = 3$, where L is the limit of this sequence, establish the value of U_0, the initial value. **3**

2. Given that $x = -2$ and $x = 1$ are two roots of the equation $x^3 + px^2 - 6x + q = 0$, establish the values of p and q and hence find the third root of the equation. **5**

3. Three vertices of the quadrilateral PQRS are P(4 , –3 , –2) Q(10 , 1 , 1) and R(7 , 4 , 3).



(a) Given that $\overrightarrow{QR} = \overrightarrow{PS}$, establish the coordinates of S. **2**

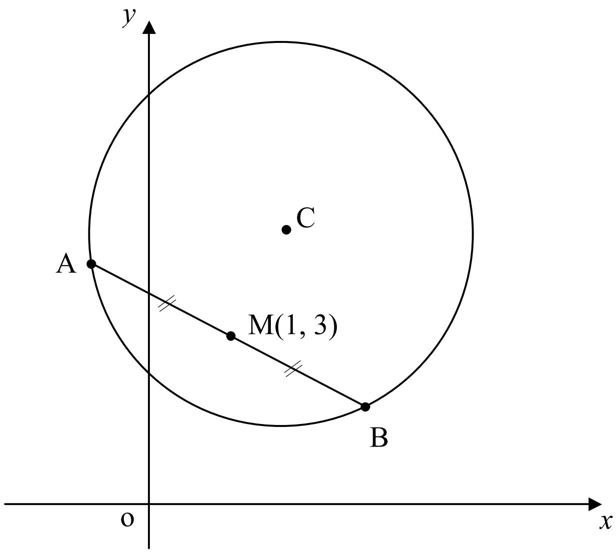
(b) Hence show that angle PSR is a right angle. **3**

4. A function is given as $f(x) = 3x^3 - 9x^2 + 27x$ and is defined on the set of real numbers.

(a) Show that the derivative of this function can be expressed in the form $f'(x) = a(x-b)^2 + c$ and write down the values of a, b and c. **4**

(b) Explain why this function has no stationary points and is in fact increasing for **all** values of x. **2**

5. The circle below, centre C, has as its equation $x^2 + y^2 - 4x - 10y + 19 = 0$.
M(1,3) is the mid-point of the chord AB.



(a) Write down the coordinates of C, the centre of the circle. **1**

(b) Show that the equation of the chord AB can be written as $x = 7 - 2y$. **3**

(c) Hence find algebraically the coordinates of A and B. **4**

6. **(a)** Given that $2\log_x y = \log_x 2y + 2$, show that x and y are related in such a way that

$$y = 2x^2.$$ **4**

(b) Hence find y when $x = \frac{1}{4}y$ and $y > 0$. **2**

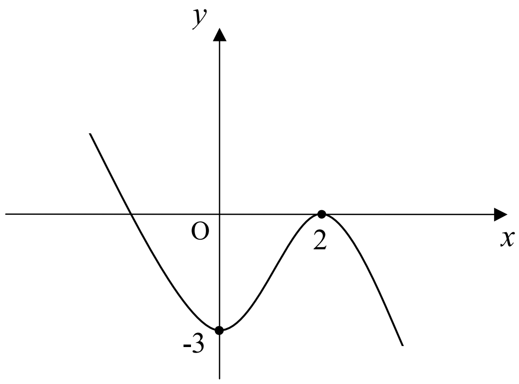
7. A function is defined as $g(\theta) = 2\cos^2\theta - 2\cos 2\theta$.

Show that $g'(\theta)$ can be written in the form

$$g'(\theta) = 2\sin 2\theta$$ **4**

8. For what value(s) of p does the equation $(4p+1)x^2 - 3px + 1 = 0$ have equal roots? **4**

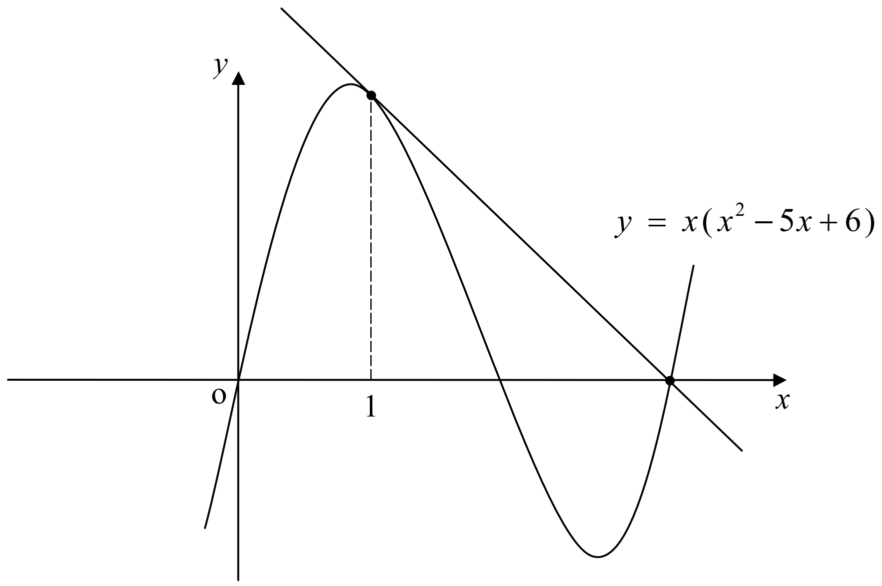
9. The diagram below shows part of the the graph of $y = g(x)$.
The function has stationary points at $(0,-3)$ and $(2,0)$ as shown.



Sketch the graph of the related function $y = g(-x) + 3$. **3**

10. Part of the graph of the curve $y = x(x^2 - 5x + 6)$ is shown in the diagram.

The tangent to the curve at the point where $x = 1$ is also shown.



(a) Find the equation of the tangent to the curve at the point where $x = 1$. **4**

(b) Show that this tangent also passes through one of the points where the curve crosses the x-axis. **2**

11. Two functions f and g are defined on the set of real numbers as follows :

$$f(x) = 8 - 2x \quad , \quad g(x) = \tfrac{1}{2}(x + 8).$$

(a) Evaluate $f\big(g(2)\big)$. **2**

(b) Find an expression , in its simplest form, for $g\big(f(x)\big)$. **2**

(c) Hence prove that $f^{-1}(x) = \tfrac{1}{2}\big[g\big(f(x)\big)\big]$. **3**

[**END OF QUESTION PAPER**]

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National Qualifications
CFE Higher Mathematics - Specimen Paper B

Mathematics
Paper 2
(Calculator)

Duration — 1 hour and 30 minutes

Total marks — 70

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\end{aligned}$$

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or

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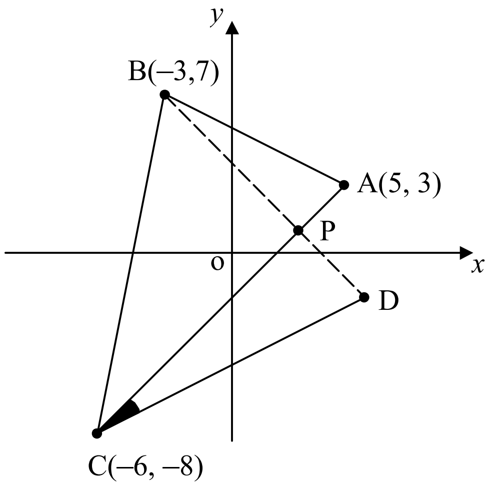
$\cos ax$	$\frac{1}{a} \sin ax + C$

Attempt ALL questions

Total marks — 70

1. Triangle ABC has vertices A(5, 3), B(–3, 7) and C(–6, –8) as shown.
The altitude through B meets AC at P.



(a) Find the equation of side AC and the equation of the altitude BP. **4**

(b) Hence find the coordinates of P. **3**

(c) BP is produced in such a way that PD = $\frac{1}{2}$BP. Establish the coordinates of D. **1**

(d) By considering gradients, calculate the size of angle DCP to the nearest degree. **3**

2. Solve algebraically the equation

$$\sin x^\circ - 3\cos 2x^\circ + 2 = 0, \qquad 0 \leq x < 360.$$ **5**

3. A function is given as $(\cos\theta - \frac{1}{2})^2 - \frac{5}{4}$ for $0 \leq \theta \leq \pi$.

State the minimum value of this function and the corresponding replacement for θ. **2**

4. Two vectors are defined as $F_1 = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $F_2 = p\underset{\sim}{i} - \sqrt{2}\,\underset{\sim}{j} + \sqrt{3}\,\underset{\sim}{k}$.

(a) Given that these two vectors have the same magnitude, find the value of p, where $p > 0$. **3**

(b) Hence calculate the angle between these two vectors, giving your answer correct to one decimal place. **3**

5. **(a)** Express $2\cos x^\circ + \sqrt{5}\sin x^\circ$ in the form $k\sin(x+\alpha)^\circ$, where k and α are constants and $k > 0$. **5**

(b) Hence state the **minimum** value of h given that $h(x) = \dfrac{12}{2\cos x^\circ + \sqrt{5}\sin x^\circ + 1}$. **1**

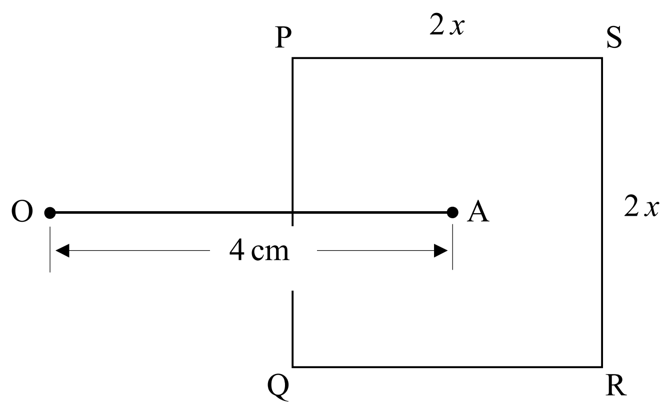
6. Evaluate $\displaystyle\int_0^1 \sqrt{(1+3x)}\ dx$ **4**

7. A radioactive substance decays according to the formula $M_t = M_o\, e^{-0{\cdot}0003t}$, where M_o is the intitial mass of the substance, M_t is the mass remaining after t years.

Calculate , **to the nearest decade**, how long a sample would take to lose 20% of its original mass. **5**

8. In the diagram below PQRS is a square of side $2x$ cm.

A straight line OA, measuring 4 cm, has been drawn in such a way that A lies at the centre of the square and OA is parallel to PS.

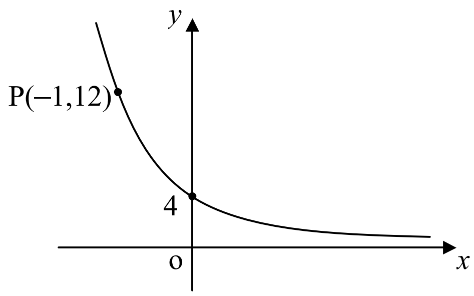


(a) Show that $\text{OP}^2 = 2x^2 - 8x + 16$. **4**

(b) Hence, by completing the square, or otherwise, find x for which the length of OP is at a minimum and state the minimum length of OP. **4**

9. Evaluate $f'(4)$ when $f(x) = \dfrac{x - 2\sqrt{x}}{x^2}$. **5**

10. The diagram shows part of the graph of $y = k(a^{-x})$.

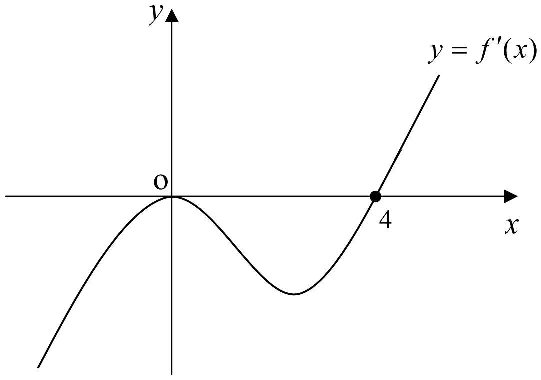


(a) Write down the value of k. **1**

(b) By considering point P, establish the value of a. **2**

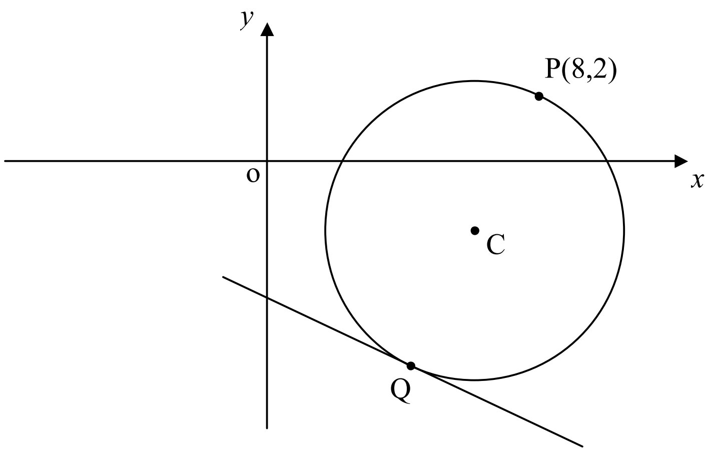
11. A function $f(x)$ has $f'(x)$ as its derivative.

Part of the graph of $y = f'(x)$ is shown below.



Sketch a possible graph for the original function, $y = f(x)$. **3**

12. The circle below, with centre C, has as its equation $x^2 + y^2 - 12x + 4y + 20 = 0$.



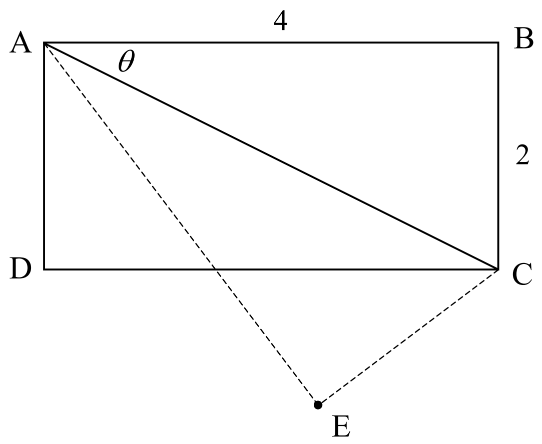
(a) Prove that the point P(8,2) does in fact lie on the circumference of this circle. **1**

(b) Hence find the equation of the tangent to the circle at the point Q, where PQ is a diameter of the circle. . **5**

13. Rectangle ABCD measures 4 units by 2 units as shown. The diagram is not to scale.

Angle BAC = θ degrees.

Point E is the reflected image of B with diagonal AC as the axis of symmetry.



(a) Show clearly that $\cos D\hat{A}E = \sin 2\theta$. **3**

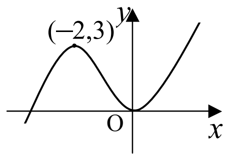
(b) Hence find the **exact** value of $\cos D\hat{A}E$. **3**

[**END OF QUESTION PAPER**]

CFE Higher Specimen Paper B – Marking Scheme Paper 1

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: statement** **(1 mark)**	
	•[1] knows condition for limit to exist	•[1] $-1 < a < 1$ (or equiv.)
(b)	**ans: 20** **(2 marks)**	
	•[1] knows how to find limit	•[1] $L = \dfrac{b}{1-a}$
	•[2] calculates limit	•[2] $L = \dfrac{8}{1-0\cdot 6} = \dfrac{8}{0\cdot 4} = \dfrac{80}{4} = 20$
(c)	**ans: $U_0 = 15$** **(3 marks)**	
	•[1] initial equating and finding U_1	•[1] $20 - U_1 = 3 \;\therefore\; U_1 = 17$
	•[2] recurrence with U_1 in place	•[2] $17 = 0\cdot 6U_0 + 8$
	•[3] answer	•[3] $9 = 0\cdot 6U_0 \Rightarrow U_0 = \dfrac{9}{0\cdot 6} = \dfrac{90}{6}$ $U_0 = 15$
2	**ans: $p = -3$, $q = 8$: $x = 4$** **(5 marks)**	
	•[1] setting up synth. division	•[1] $1 \,\big\vert\; 1 \quad p \quad -6 \quad q$
	•[2] obtaining first equation	•[2] $p + q = 5$
	•[3] obtaining second equation	•[3] $4p + q = -4$
	•[4] solving system for p and q	•[4] $p = -3$, $q = 8$
	•[5] sub. for 3rd root	•[5] $x^2 - 2x - 8 = 0 \Rightarrow (x+2)(x-4) = 0$ $x = 4$ is missing root
3(a)	**ans:** S(1,0,0) **(2 marks)**	
	•[1] finding displacement $\vec{QR}$	•[1] $\vec{QR} = r - a = \begin{pmatrix}7\\4\\3\end{pmatrix} - \begin{pmatrix}10\\1\\1\end{pmatrix} = \begin{pmatrix}-3\\3\\2\end{pmatrix}$
	•[2] establishing coordinates of S	•[2] $s = \begin{pmatrix}4\\-3\\-2\end{pmatrix} + \begin{pmatrix}-3\\3\\2\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ S(1, 0, 0)
(b)	**ans:** proof **(3 marks)**	
	•[1] knowing $\vec{SR}.\vec{SP} = 0$, for R.A. (stated or implied)	•[1] For right-angle $\vec{SR}.\vec{SP} = 0$
	•[2] For both displacements	•[2] $\vec{SR}.\vec{SP} = \begin{pmatrix}6\\4\\3\end{pmatrix}\begin{pmatrix}3\\-3\\-2\end{pmatrix} = \ldots\ldots\ldots\ldots$
	•[3] scalar product calculation to zero	•[3] $= 18 - 12 - 6 = 0$; therefore right-angled

	Give 1 mark for each •	Illustration(s) for awarding each mark
4(a)	**ans: $a=9$, $b=1$, $c=18$** **(4 marks)**	
	•[1] differentiating	•[1] $f'(x)=9x^2-18x+27$
	•[2] common factor	•[2] $9(x^2-2x)+27$
	•[3] completes the square	•[3] $9[(x-1)^2-1]+27=9(x-1)^2+18$
	•[4] lists a, b and c	•[4] $a=9$, $b=1$, $c=18$
(b)	**ans:** explanation **(2 marks)**	
	•[1] knows condition for increasing function	•[1] $f'(x)>0$ for increasing function
	•[2] explanation	•[2] min value of $(x-1)^2+18$ = 18 so always +ve , always incr.
5(a)	**ans: C(2,5)** **(1 mark)**	
	•[1] answer	•[1] C(2,5)
(b)	**ans: proof** **(3 marks)**	
	•[1] gradient of CM	•[1] $m_{cm}=\frac{5-3}{2-1}=2$
	•[2] finds perp. gradient of chord AB	•[2] $m_{AB}=-\frac{1}{2}$
	•[3] subs into straight line equation and rearranges to answer	•[3] $y-3=-\frac{1}{2}(x-1)$ $x=7-2y$
(c)	**ans: A(–1, 4) , B(3, 2)** **(4 marks)**	
	•[1] subs line into circle equation	•[1] $(7-2y)^2+y^2-4(7-2y)-10y+19=0$
	•[2] expanding and simplifying	•[2] $5y^2-30y+40=0$
	•[3] factorising and finding y coords	•[3] $(5(y-4)(y-2)-0 \;\therefore\; y=4$, $y=2$
	•[4] stating coordinates of A and B	•[4] A(–1, 4) , B(3, 2)
6(a)	**ans: *proof*** **(4 marks)**	
	•[1] logs to same side	•[1] $2\log_x y-\log_x 2y=2$
	•[2] moving the power	•[2] $\log_x y^2-\log_x 2y=2$
	•[3] combining logs	•[3] $\log_x \frac{y^2}{2y}=2$
	•[4] changing to index form	•[4] $x^2=\frac{1}{2}y \Rightarrow y=2x^2$
(b)	**ans:** $y=8$ **(2 marks)**	
	•[1] substitution	•[1] $y=2(\frac{1}{4}y)^2$
	•[2] manipulation and answer	•[2] $y=8$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7	**ans: proof** **(5 marks)** •1 differentiates first term •2 differentiates second term •3 extracting $2\sin\theta\cos\theta$ for replacing •4 simplifying to given answer	 •1 $-4\cos\theta\sin\theta$ ….. •2 ….($-4\sin 2\theta$) •3 $-2(2\sin\theta\cos\theta)-(-4\sin 2\theta)$ •4 $-2\sin 2\theta+4\sin 2\theta=2\sin 2\theta$
8	**ans:** $\boldsymbol{p=-\frac{2}{9}}$ **,** $\boldsymbol{p=2}$ **(4 marks)** •1 discr. = 0 (stated or implied) •2 selecting a, b and c •3 substituting and simplifying •4 factorising to answers	 •1 for equal roots $b^2-4ac=0$ •2 $a=4p+1$, $b=-3p$, $c=1$ •3 $(-3p)^2-(4(4p+1).1)=0$ $\Rightarrow 9p^2-16p-4=0$ •4 $(9p+2)(p-2)=0 \Rightarrow p=-\frac{2}{9}$ *or* 2
9	**ans: diagram** **(3 marks)** •1 reflection in y-axis •2 translating …. 3 up •3 annotating final sketch	
10(a) **(b)**	**ans:** $y=-x+3$ **(4 marks)** •1 completing point of tangency •2 differentiating to find m •3 finding gradient of tangent •4 point + m in equation **ans:** $\boldsymbol{2y-x=13}$ **(2 marks)** •1 Finding where tan. cuts x-axis •2 Showing that point satisfies equation	 •1 $y=1(1-5+6)=2 \therefore T(1,\ 2)$ •2 $\frac{dy}{dx}=m=3x^2-10x+6$ •3 $m=3(1^2)-10(1)+6=-1$ •4 $y-2=-1(x-1)$ •1 When $y=0$ then $x=3$ •2 Sub $x=3$ in equ. of curve $y=3(3^2-5(3)+6)=3(0)=0$

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11(a)	**ans:** $f(g(2)) = -2$ **(2 marks)**	
	$\bullet^1$ finds $g(2)$ $\bullet^2$ finds $f(5)$	$\bullet^1$ $g(2) = 5$ $\bullet^2$ $f(5) = -2$
(b)	**ans:** $g(f(x)) = 8 - x$ **(2 marks)**	
	$\bullet^1$ substitution $\bullet^2$ simplifying	$\bullet^1$ $g(8-2x) = \frac{1}{2}\left((8-2x)+8\right)$ $\bullet^2$ $g(f(x)) = \frac{1}{2}(16-2x) = 8 - x$
(c)	**ans: proof** **(3 marks)**	
	$\bullet^1$ knowing how to find inverse $\bullet^2$ finding inverse $\bullet^3$ final statement	$\bullet^1$ $f^{-1}(x) \Rightarrow y = 8 - 2x$ $\bullet^2$ $2x = 8 - y \Rightarrow x = \frac{1}{2}(8-y)$ $\bullet^3$ $f^{-1}(x) = \frac{1}{2}(8-x) = \frac{1}{2}[g(f(x))]$
		Total: 60 marks

CFE Higher Specimen Paper B – Marking Scheme Paper 2

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans:** $y = x - 2$, $y = -x + 4$ **(4 marks)**	
	•1 gradient of AC	•1 $m_{AC} = \frac{3+8}{5+6} = 1$
	•2 equation of AC	•2 $y - 3 = 1(x - 5)$
	•3 finding gradient of altitude	•3 $m_{alt} = -1$
	•4 equation of altitude	•4 $y - 7 = -1(x + 3)$
(b)	**ans:** P(3, 1) **(3 marks)**	
	•1 knowing to solve a system	•1 $x - 2 = -x + 4$
	•2 finding first coordinate	•2 $2x = 6 \Rightarrow x = 3$
	•3 finding second coordinate	•3 $y = 3 - 2 = 1$
(c)	**ans:** D(6, –2) **(1 mark)**	
	•1 answer	•1 $\rightarrow 6 \downarrow 6 \therefore \rightarrow 3 \downarrow 3$ from P, D(6, –2)
(d)	**ans:** 18° **(3 marks)**	
	•1 knowing and using $\tan\theta = m$	•1 $m_{AC} = 1 \therefore \tan^{-1} 1 = \theta = 45^\circ$
	•2 angle between CD and horizontal	•2 $m_{CD} = \frac{-2+8}{6+6} = 0\cdot5 \therefore \tan^{-1} 0\cdot5 = 26\cdot6^\circ$
	•3 45° and subtraction to ans.	•3 $45 - 26\cdot6 = 18^\circ$
2	**ans:** $19\cdot5^\circ$, $160\cdot5^\circ$, 210° , 330° **(5 marks)**	
	•1 correct substitution	•1 $\sin x - 3(1 - 2\sin^2 x) + 2 = 0$
	•2 re-arranging to quadratic	•2 $6\sin^2 x + \sin x - 1 = 0$
	•3 factorising to two roots	•3 $\sin x = \frac{1}{3}$ *or* $\sin x = -\frac{1}{2}$
	•4 two ans. from one root	•4 $19\cdot5^o$, $160\cdot5^o$
	•5 two ans. from second root	•5 210^o , 330^o
3	**ans: min** $= -\frac{5}{4}$ **at** $\boldsymbol{\theta} = \frac{\pi}{3}$ **(2 marks)**	
	•1 states minimum value	•1 minimum $= -\frac{5}{4}$
	•2 corresponding value for θ	•2 $\theta = \frac{\pi}{3}$

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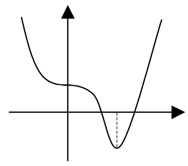
4(a)	**ans:** $p = 2$ **(3 marks)**	
	•¹ calculating magnitude of F_1	•¹ $\lvert F_1 \rvert = \sqrt{2^2 + 2^2 + (-1)^2} = 3$
	•² equating second magnitude	•² $\lvert F_2 \rvert$ then $p^2 + 2 + 3 = 9$ (or equiv.)
	•³ answer	•³ $p = 2$
(b)	**ans:** $\theta = 93 \cdot 6^\circ$ **(3 marks)**	
	•¹ scalar product of F_1 and F_2	•¹ $F_1.F_2 = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} . \begin{pmatrix} 2 \\ -\sqrt{2} \\ \sqrt{3} \end{pmatrix} = 4 - 2\sqrt{2} - \sqrt{3}$
	•² substitution	•² $\cos\theta = \dfrac{4 - 2\sqrt{2} - \sqrt{3}}{3 \times 3}$
	•³ answer	•³ $\theta = 93 \cdot 6^\circ$
5(a)	**ans:** $\mathbf{3\sin(x + 41 \cdot 8)^\circ}$ **(5 marks)**	
	•¹ correct expansion	•¹ $= k\sin x\cos\alpha + k\cos x\sin\alpha$
	•² equating coefficients	•² $k\cos\alpha = \sqrt{5}$, $k\sin\alpha = 2$
	•³ tan ratio + correct quadrant	•³ $\tan\alpha = \dfrac{2}{\sqrt{5}}$, 1st quadrant
	•⁴ finding k	•⁴ $k = \sqrt{9} = 3$
	•⁵ finding α	•⁵ $\alpha = 41 \cdot 8^\circ$
(b)	**ans: 3** **(1 mark)**	
	•¹ answer	•¹ max $= 3$, $h_{max} = \dfrac{12}{3+1} = 3$
6	**ans:** $\frac{14}{9}$ **(4 marks)**	
	•¹ new power	•¹ $......(1+3x)^{\frac{3}{2}}$
	•² front term	•² $\dfrac{1}{\frac{3}{2} \times 3}$ .... or equivalent
	•³ substitution	•³ $[\frac{2}{9}(1+3(1))^{\frac{3}{2}}] - [\frac{2}{9}(1+0)^{\frac{3}{2}}]$
	•⁴ calculation to answer	•⁴ $\frac{14}{9}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7	**ans: 740 years** **(5 marks)**	
	•[1] solving exponent to $0\cdot8$	•[1] $e^{-0\cdot0003t} = 0\cdot8$
	•[2] taking logs of both sides	•[2] $\log_e e^{-0\cdot0003t} = \log_e 0\cdot8$
	•[3] releasing power to front	•[3] $-0\cdot0003t\log_e e = \log_e 0\cdot8$
	•[4] making t the subject	•[4] $-0\cdot0003t = \log_e 0\cdot8$ $t = \dfrac{\log_e 0\cdot8}{-0\cdot0003}$
	•[5] answer (ignore rounding)	•[5] 740 years $(743\cdot8)$
8(a)	**ans:** proof **(4 marks)**	
	•[1] attempting to use Pythagoras	•[1] $\text{OP}^2 = a^2 + b^2$ (stated or implied)
	•[2] length x	•[2] $\text{OP}^2 = x^2 + \ldots\ldots\ldots\ldots$
	•[3] length $(4-x)$	•[3] $\text{OP}^2 = \ldots\ldots\ldots + (4-x)^2$
	•[4] expansion to answer	•[4] $\text{OP}^2 = x^2 + 16 - 8x + x^2$ $= 2x^2 - 8x + 16$
(b)	**ans:** $x = 2$, $\text{OP}_{\min} = \sqrt{8}$ **(4 marks)**	
	•[1] removing common factor	•[1] $2(x^2 - 4x) + 16$
	•[2] completing the square with $x^2 - 4x$	•[2] $[(x-2)^2 - 4]$
	•[3] tidying to final form	•[3] $\text{OP}^2 = 2(x-2)^2 + 8$
	•[4] answer for replacement and minimum	•[4] minimum when x = 2 minimum value of $\text{OP}^2 = 8$ $\therefore \text{OP}_{\min} = \sqrt{8}$
9	**ans:** $f'(4) = \dfrac{1}{32}$ **(5 marks)**	
	•[1] preparing to differentiate	•[1] $f(x) = x^{-2}(x - 2x^{\frac{1}{2}})$ $= x^{-1} - 2x^{-\frac{3}{2}}$
	•[2] differentiating first term	•[2] $f'(x) = -x^{-2}\ \ldots\ldots\ldots\ldots$
	•[3] differentiating second term	•[3] $f'(x) = \ldots\ldots\ldots\ldots\ 3x^{-\frac{5}{2}}$
	•[4] subst. $x = 4$ in derivative	•[4] $f'(x) = -\dfrac{1}{4^2} + \dfrac{3}{4^{\frac{5}{2}}}$
	•[5] answer	•[5] $f'(4) = -\dfrac{2}{32} + \dfrac{3}{32} = \dfrac{1}{32}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
10(a)	**ans: $k = 4$** **(1 mark)**	
	•[1] answer	•[1] $k = 4$
(b)	**ans: $a = 3$** **(2 marks)**	
	•[1] substituting in point & k •[2] calculation and answer	•[1] $y = 4(a^{-x}) \Rightarrow 4(a^{-(-1)}) = 12$ •[2] $4a = 12$ $a = 3$
11	**ans:** see sketch opposite **(3 marks)**	
	•[1] downward point of inflexion on y-axis (anywhere on y-axis including the origin) •[2] for minimum T.P. •[3] for 4 marked on x-axis @ min. T.P.	possible example … 
12(a)	**ans: proof** **(1 mark)**	
	•[1] Substituting to prove	•[1] $8^2 + 2^2 - 12(8) + 4(2) + 20 = 0$
(b)	**ans:** $2y = -x - 8$ **(5 marks)**	
	•[1] centre •[2] coords. of Q •[3] gradient of radius •[4] gradient of tangent •[5] point + grad. in equ. to answer	•[1] C(6, – 2) •[2] Q(4, –6) ... by stepping out or equiv. •[3] $m_r = \frac{-2-(-6)}{6-4} = 2$ •[4] $m_{\tan} = -\frac{1}{2}$ •[5] $y + 6 = -\frac{1}{2}(x - 4)$
13(a)	**ans: proof** **(3 marks)**	
	•[1] for realising $\angle$DAE = $90 - 2\theta$ •[2] using compound angle replacement •[3] exact values to answer	•[1] $2\theta + \angle DAE = 90$ (stated or implied) •[2] $\cos DAE = \cos(90 - 2\theta)$ $= \cos 90 \cos 2\theta + \sin 90 \sin 2\theta$ •[3] $= (0) \times \cos 2\theta + (1) \times \sin 2\theta$ $= \sin 2\theta$
(b)	**ans:** $\frac{4}{5}$ (accept $\frac{16}{20}$) **(3 marks)**	
	•[1] for using $\sin 2\theta$ and using replacement •[2] calculating hypotenuse •[3] substituting exact values then answer	•[1] $\sin 2\theta = 2 \sin\theta \cos\theta$ •[2] $AC = \sqrt{4^2 + 2^2} = \sqrt{20}$ (or equiv.) •[3] $\sin 2\theta = 2 \times \frac{2}{\sqrt{20}} \times \frac{4}{\sqrt{20}} = \frac{16}{20} = \frac{4}{5}$
		Total: 70 marks