H

National Qualifications
CFE Higher Mathematics - Specimen Paper C

Mathematics
Paper 1
(Non-Calculator)

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

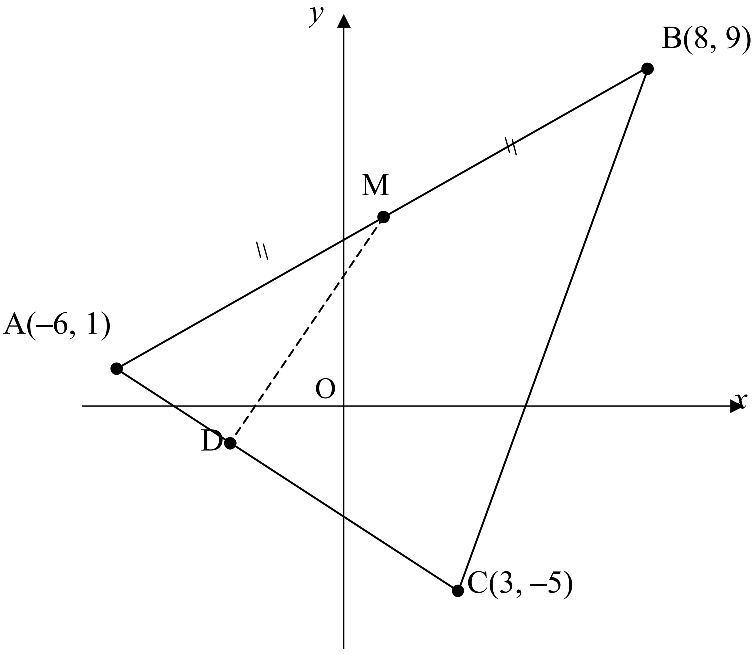
$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Attempt ALL questions

Total marks — 60

1. Triangle ABC has vertices A(–6, 1), B(8, 9) and C(3, –5) as shown.

M is the mid-point of side AB and D is a point on side AC.



(a) Write down the coordinates of M. **1**

(b) Find the equation of MD given that MD is **perpendicular** to side AC. **4**

(c) Hence establish the coordinates of D. **5**

2. Solve the equation $3\cos 2\theta + 1 = 5\sin\theta$, for $0 < \theta < \pi$. **6**

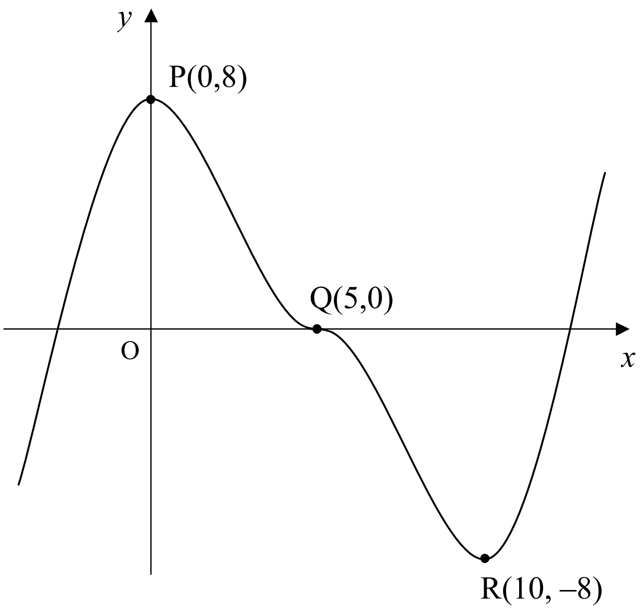
3. Given that $\log_2(x+3) = 2\log_2 x + 2$, find x if $x > 0$. **5**

4. A curve has as its equation $y = (p+1)x^3 - 3px^2 + 4x + 1$, where p **is a positive integer**.

(a) Find $\dfrac{dy}{dx}$. **2**

(b) Hence establish the value of p given that this curve has only **one stationary point**. **5**

5. The diagram shows part of the graph of $y = f(x)$.



The function has stationary points at P(0, 8), Q(5, 0) and R(10, –8) as shown.

Sketch a possible graph for $y = f'(x)$, where $f'(x)$ is the derivative of $f(x)$. **4**

6. Two vectors are defined as $V_1 = \sqrt{6}\,\boldsymbol{i} + \boldsymbol{j} + \sqrt{8}\,\boldsymbol{k}$ and $V_2 = 4\,\boldsymbol{i} + \sqrt{24}\,\boldsymbol{j} + a\sqrt{3}\,\boldsymbol{k}$, where a is a constant and all coefficients of $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are greater than zero.

Given that the vectors V_1 and V_2 are perpendicular, calculate the value of a. **4**

7. A recurrence relation is defined as $U_{n+1} = aU_n + b$, where a and b are constants.

Given that $U_1 = 100$, $U_2 = 80$ and $U_3 = 66$, form a system of equations and solve it to find the values of the constants a and b. **3**

8. Two functions are defined on suitable domains as $f(x) = 2 + \dfrac{1}{x}$ and $g(x) = x^2 - 2x$.

(a) Evaluate $g\left(f(\tfrac{1}{2})\right)$. **2**

(b) Show clearly that the composite function $g\left(f(a)\right)$ can be expressed in the form

$$g\left(f(a)\right) = \frac{2a + 1}{a^2}.$$ **4**

9. Solve the equation $x^3 - 4x^2 + x + 6 = 0$. **4**

10. A function is defined on a suitable domain as $f(x) = \dfrac{25}{2x - 1}$.

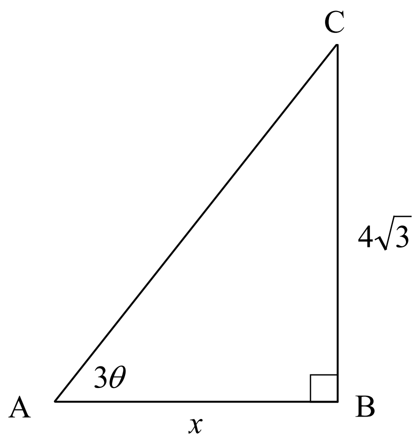
For what values of x is the rate of change of this function equal to -2? **5**

11. Consider the triangle opposite.

AB is x units long, BC $= 4\sqrt{3}$ units long and angle BAC $= 3\theta$ radians.

(a) Given that the exact area of the triangle is $8\sqrt{3}$ units^2, **show clearly** that $x = 4$. **3**

(b) Hence find the value of θ, **in radians**, given that 3θ is acute. **3**



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National Qualifications
CFE Higher Mathematics - Specimen Paper C

Mathematics
Paper 2
(Calculator)

Duration — 1 hour and 30 minutes

Total marks — 70

Attempt ALL questions.

You may use a calculator.

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State the units for your answer where appropriate.

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Attempt ALL questions

Total marks — 70

1. **(a)** Show that the equation $m(4x+1)(x+1)+4x+5=0$ can be written in the form

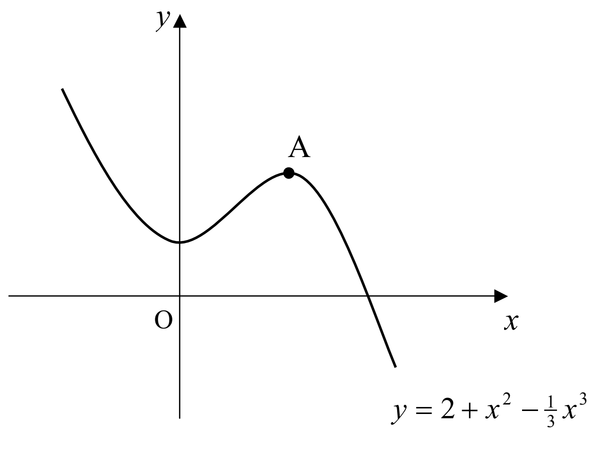
$$4mx^2 + (5m+4)x + (m+5) = 0$$ **2**

(b) Hence find m, where $m>1$, given that the equation has equal roots. **4**

2. **(a)** Express $\sqrt{5}\cos x^\circ - 2\sin x^\circ$ in the form $k\cos(x+a)^\circ$ for $0<a<360$. **4**

(b) Hence state the maximum value of $\sqrt{5}\cos x^\circ - 2\sin x^\circ$ and the value of x at which this maximum occurs. **2**

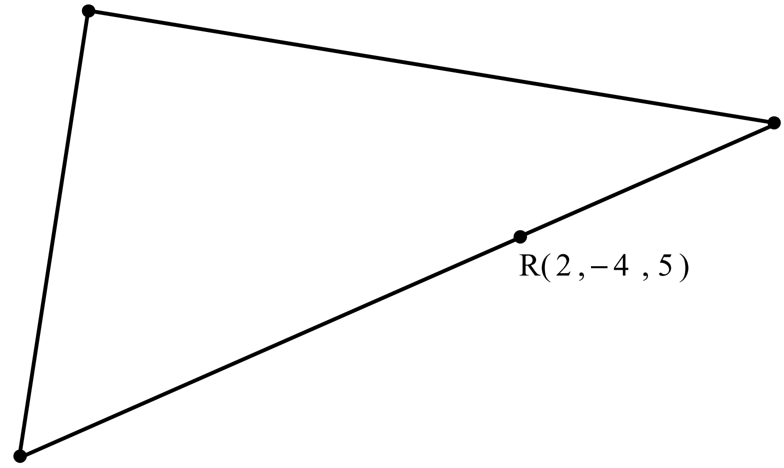
3. Part of the graph of $y = 2 + x^2 - \frac{1}{3}x^3$ is shown below.



Find the coordinates of the point A. **5**

4. P, Q and R are the points $(-2\ ,-2\ ,1)$, $(4\ ,-5\ ,7)$ and $(2\ ,-4\ ,5)$ respectively.

S has coordinates $(1\ ,1\ ,4)$ as shown.



(a) Show clearly that P, Q and R are collinear and find the ratio $\frac{\text{PR}}{\text{RQ}}$. **3**

(b) Show that $\cos \text{SPR} = \frac{1}{\sqrt{3}}$ **4**

5. Find the equation of the tangent to the curve $y = x^3 - 3x$ at the point where $x = 2$. **5**

6. Express the function $f(x) = 3x^2 - 6x + 11$ in the form $p(x-q)^2 + r$. **3**

7. Water flows from a small reservoir according to the law $h_t = h_o e^{-kt}$, where h_o is the original height of the water level and h_t the height of the water after t days.

(a) If the water level decreases by **one fifth** of its original height in 10 days, calculate the value of k correct to 2 significant figures. **3**

(b) Calculate the number of days required to reduce the water level to **half its original height**. **3**

8. Given that $f(\frac{\pi}{6}) = -2$ and $f'(x) = -8\sin 4x$, find an expression for $f(x)$. **4**

9. Certain radioisotopes are used as *tracers*, to track down diseased tissue within the body, and then be absorbed, to act as a long-term radio-therapy treatment. Their passage through the body and mass is ascertained by means of a Geiger-Müller counter.

During trials of a particular radioisotope the following information was obtained.

- *the isotope loses* 3% *of its mass every hour*
- *the maximum recommended mass in the bloodstream is* 165mgs
- 100mgs *is the smallest mass detectable by the Geiger-Müller counter*

(a) An intial dose of 150mgs of the isotope is injected into a patient.

Would the mass remaining after 12 hours still be detectable by the Geiger-Müller counter? **3**

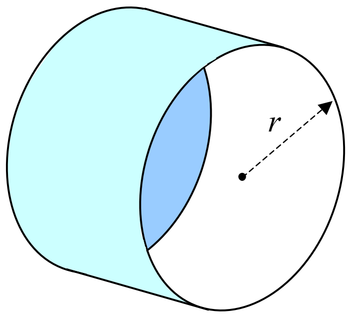
Your answer must be accompanied by appropriate working.

(b) **After the initial dose**, top-up injections of 50mgs are given every 12 hours.

Comment on the long-term suitabilty of this plan. **4**

Your answer must be accompanied by appropriate working.

10. A small open cylindrical glass container has a radius of $r$cm as shown in the diagram.

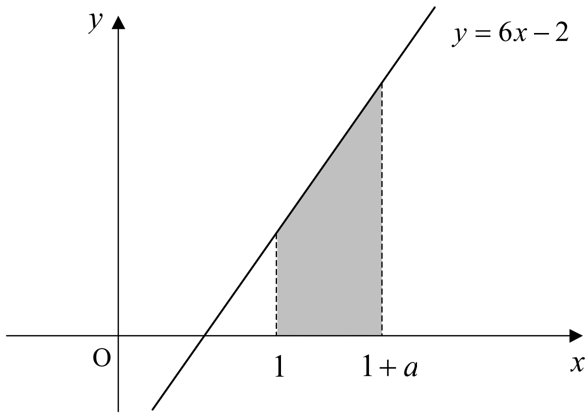


The **total surface area** (A), expressed in terms of r, is found to be

$$A(r) = \frac{120}{r} + \pi r^2.$$

Find the radius of the cylinder so that the surface area (A) is at a minimum.
Give your answer correct to 2-decimal places. **5**

11. The diagram below, which is not to scale, shows part of the graph of the line with equation $y = 6x - 2$. Also shown are ordinates at $x = 1$ and at $x = 1 + a$.

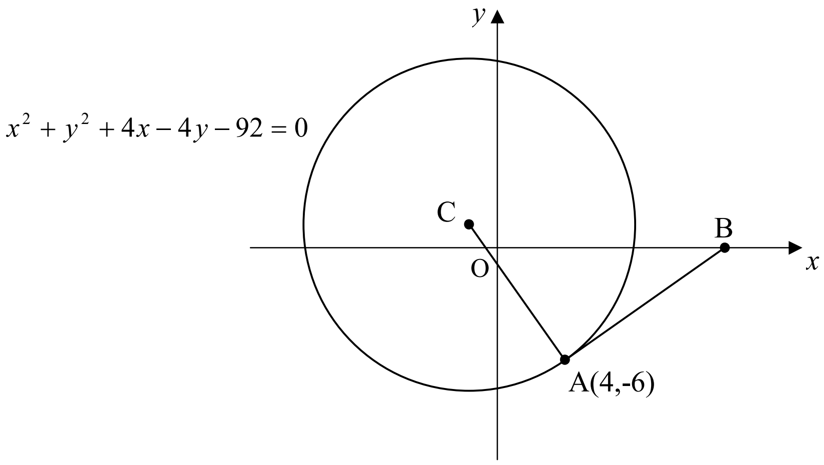


Using calculus, find a given that the shaded part of the diagram has an area of 4 square units. **7**

12. A circle, centre C, has as its equation $x^2 + y^2 + 4x - 4y - 92 = 0$.

The point A(4,-6) lies on the circumference of the circle.

The line AB is a tangent to the circle with B lying on the x-axis as shown.



(a) Find the equation of the line AB. **(4)**

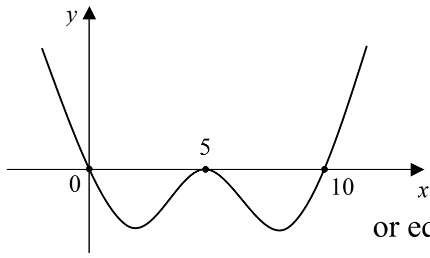
(b) Hence write down the coordinates of B. **(2)**

(c) Establish the equation of the circle passing through the points A, B and C. **(3)**

[**END OF QUESTION PAPER**]

CFE Higher Specimen Paper C – Marking Scheme Paper 1

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans:** M(1, 5) **(1 mark)**	
	•[1] answer	•[1] M(1, 5)
(b)	**ans:** $2y = 3x + 7$ **(4 marks)**	
	•[1] gradient of AC	•[1] $m_{AC} = \frac{-5-1}{3+6} = -\frac{2}{3}$
	•[2] perpendicular gradient	•[2] $m_{MD} = \frac{3}{2}$
	•[3] substitution	•[3] $y - 5 = \frac{3}{2}(x-1)$
	•[4] equation (any form)	•[4] $2y = 3x + 7$ (or equiv)
(c)	**ans:** D(–3, –1) **(5 marks)**	
	•[1] for strategy (system of equ's)	•[1] using a system
	•[2] equation of AC	•[2] $y - 1 = -\frac{2}{3}(x+6)$
	•[3] multiplying to equation	•[3] $3y = -2x - 9$ (or equiv.)
	•[4] finding first coordinate	•[4] $y = -1$
	•[5] finding second coordinate	•[5] $2(-1) = 3x + 7 \therefore x = -3$
2	**ans:** $\theta = \frac{\pi}{6}$ *or* $\frac{5\pi}{6}$ **(6 marks)**	
	•[1] for double angle substitution	•[1] $3(1 - 2\sin^2\theta) + 1 = 5\sin\theta$
	•[2] solving to zero	•[2] $6\sin^2\theta + 5\sin\theta - 4 = 0$
	•[3] factorising	•[3] $(3\sin\theta + 4)(2\sin\theta - 1) = 0$
	•[4] solving	•[4] $\sin\theta = \frac{-4}{3}$ *or* $\sin\theta = \frac{1}{2}$
	•[5] discarding one root (no solution)	•[5] $\sin\theta = \frac{-4}{3}$ discarded no solution from $\sin\theta = \frac{1}{2}$
	•[6] finding correct answers in radians	•[6] $\theta = \frac{\pi}{6}$ *or* $\frac{5\pi}{6}$
3	**ans:** $x = 1$ **(5 marks)**	
	•[1] making 2 a power	•[1] $\log_2(x+3) = \log_2 x^2 + 2$
	•[2] taking logs to the one side	•[2] $\log_2(x+3) - \log_2 x^2 = 2$
	•[3] combining the logs	•[3] $\log_2\left(\frac{x+3}{x^2}\right) = 2$
	•[4] converting to index form	•[4] $2^2 = \frac{x+3}{x^2}$
	•[5] solving quadratic equation to answer	•[5] $4x^2 - x - 3 = 0$ $(4x+3)(x-1) = 0$ $x = -\frac{3}{4}$ *or* 1

	Give 1 mark for each •	Illustration(s) for awarding each mark
4(a)	**ans:** $\frac{dy}{dx} = 3(p+1)x^2 - 6px + 4$ **(2 marks)**	
	•[1] differentiating first term •[2] differentiating remainder	•[1] $\frac{dy}{dx} = 3(p+1)x^2$ .... (or equiv.) •[2] $\frac{dy}{dx} =$ .... $-6px + 4$
(b)	**ans:** $p = 2$ **(5 marks)**	
	•[1] realising strategy i.e. equal roots •[2] for a, b and c •[3] for substitution •[4] for simplifying + factorising •[5] choosing correct answer	•[1] $b^2 - 4ac = 0$ (stated <u>or</u> implied) •[2] $a = 3p+3,\ b = -6p,\ c = 4$ •[3] $(-6p)^2 - 16(3p+3) = 0$ •[4] $36p^2 - 48p - 48 = 0$ $12(3p+2)(p-2) = 0$ •[5] $\therefore\ \cancel{p = -\frac{2}{3}}\ ,\ p = 2$
5	**ans:** see sketch **(4 marks)** •[1] stat. points as roots •[2] basic shape … left side •[3] basic shape … right side •[4] annotation	 or equivalent sketch
6	**ans:** $a = -3$ **(4 marks)**	
	•[1] for column vectors	•[1] $V_1 = \begin{pmatrix} \sqrt{6} \\ 1 \\ \sqrt{8} \end{pmatrix},\ V_2 = \begin{pmatrix} 4 \\ \sqrt{24} \\ a\sqrt{3} \end{pmatrix}$
	•[2] for scalar product strategy (stated or implied) •[3] calculating scalar product •[4] surd manipulation to answer	•[2] If perp. than $V_1 . V_2 = 0$ •[3] $V_1 . V_2 = 4\sqrt{6} + \sqrt{24} + a\sqrt{24} = 0$ •[4] $4\sqrt{6} + 2\sqrt{6} + 2a\sqrt{6} = 0$ $2a = -6 \quad \rightarrow \quad a = -3$
7	**ans:** $a = 0{\cdot}7$ **and** $b = 10$ **(3 marks)**	
	•[1] for constructing equations •[2] solving for a •[3] finding b	•[1] $80 = 100a + b$ $66 = 80a + b$ •[2] $14 = 20a \quad \therefore\ a = \frac{14}{20} = \frac{7}{10} = 0 \cdot 7$ •[3] $80 = 100(0{\cdot}7) + b \quad \therefore\ b = 10$

	Give 1 mark for each	Illustration(s) for awarding each mark
8(a)	**ans: 8** **(2 marks)**	
	●[1] correct answer from first function	●[1] $f(\frac{1}{2}) = 2 + \frac{1}{\frac{1}{2}} = 2 + 2 = 4$
	●[2] correct answer from second function	●[2] $g(4) = 16 - 2(4) = 8$
(b)	**ans: proof** **(4 marks)**	
	●[1] sub in function correctly (or equiv.)	●[1] $g(2+\frac{1}{a}) = (2+\frac{1}{a})^2 - 2(2+\frac{1}{a})$
	●[2] expanding brackets	●[2] $4 + \frac{4}{a} + \frac{1}{a^2} - 4 - \frac{2}{a}$
	●[3] simplifying	●[3] $\frac{2}{a} + \frac{1}{a^2}$
	●[4] common denom. to correct form	●[4] $\frac{2a}{a^2} + \frac{1}{a^2}$ …… answer
9	**ans:** $x = -1$, $x = 2$, $x = 3$ **(4 marks)**	
	●[1] setting up synthetic division	●[1] $\begin{array}{c\|cccc} n & 1 & -4 & 1 & 6 \\ \hline & & & & \end{array}$
	●[2] attempting to find first root (rem = 0)	●[2] $\begin{array}{c\|cccc} 2 & 1 & -4 & 1 & 6 \\ & & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \end{array}$
	●[3] using the quotient to find remaining roots	●[3] $x^2 - 2x - 3 = 0$ (or equiv.)
	●[4] correct answer if ans. left as $(x+1)(x-2)(x-3) = 0$, ¾ *marks*	●[4] $x = -1$, $x = 2$, $x = 3$
10	**ans:** $x = -2$ **or 3** **(5 marks)**	
	●[1] for preparing to differentiate	●[1] $f(x) = 25(2x-1)^{-1}$
	●[2] first differentiation	●[2] $f'(x) = -1 \times 25(2x-1)^{-2}$ ...... (or equiv.)
	●[3] differentiating inside brackets	●[3] $f'(x) =$ .... $\times 2$
	●[4] equating to –2	●[4] $\dfrac{-50}{(2x-1)^2} = -2$
	●[5] solving	●[5] $(2x-1)^2 = 25$ (or equiv.) $2x - 1 = \pm\sqrt{25}$ $x = -2$ or 3

	Give 1 mark for each	Illustration(s) for awarding each mark
11	**(a) ans: proof** **(3 marks)**	
	●¹ area strategy	●¹ $A = \frac{1}{2}bh$
	●² substitution	●² $A = \frac{1}{2}bh = \frac{1}{2} \times x \times 4\sqrt{3}$
	●³ answer	●³ $8\sqrt{3} = x \times 2\sqrt{3} \quad \therefore \; x = 4$
	(b) ans: $\boldsymbol{\theta = \frac{\pi}{9}}$ **(3 marks)**	
	●¹ strategy and writing .. $\tan 3\theta =$	●¹ $\tan 3\theta = \dfrac{4\sqrt{3}}{4} = \sqrt{3}$
	●² knowing exact value	●² if $\tan 3\theta = \sqrt{3}$ then $3\theta = \frac{\pi}{3}$
	●³ calculating answer	●³ $\therefore \; \theta = \frac{\pi}{9}$
		Total: 60 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: proof** **(2 marks)**	
	•[1] expanding the brackets •[2] express in suitable form	•[1] $4mx^2 + 5mx + m + 4x + 5$ •[2] $4mx^2 + (5m+4)x + (m+5)$
(b)	**ans:** $m = 4$ **(4 marks)**	
	•[1] know to use use discriminant = 0 •[2] select correct a, b and c •[3] simplify discriminant •[4] solve for m *(discard other root)*	•[1] for equal roots $b^2 - 4ac = 0$ (stated or implied) •[2] $a = 4m$, $b = 5m + 4$, $c = m + 5$ •[3] $(5m+4)^2 - 16m(m+5) = 0$ $9m^2 - 40m + 16 = 0$ •[4] $(9m-4)(m-4) = 0$ $\therefore \cancel{m = \frac{4}{9}}$ *or* $m = 4$
2(a)	**ans:** $3\cos(x + 41{\cdot}8)^{o}$ **(4 marks)**	
	•[1] expands and matches coefficients •[2] finds value of k •[3] finds value of a •[4] writes in required form	•[1] $k\cos(x+a)° = k\cos x°\cos a° - k\sin x°\sin a°$ $k\cos a^{o} = \sqrt{5}$; $k\sin a^{o} = 2$ •[2] $k = \sqrt{(\sqrt{5}^{2} + 2^2} = \sqrt{9} = 3$ •[3] $\tan a° = \frac{2}{\sqrt{5}}; a = 41 \cdot 8°$ •[4] $3(x + 41 \cdot 8)°$
(b)	**ans:** **3; 318·2°** **(2 marks)**	
	•[1] states maximum value •[2] finds value of x	•[1] maximum value = 3 •[2] $\cos(x + 41 \cdot 8)° = 1$; $x = 318{\cdot}2^{o}$
3	**ans:** $\mathbf{A(2, 3\frac{1}{3})}$ **(5 marks)**	
	•[1] knows derivative equal to zero •[2] takes derivative •[3] solves for x •[4] substitutes for x and finds y •[5] states the coordinates of A	•[1] $\frac{dy}{dx} = 0$ [stated or implied] •[2] $2x - x^2 = 0$ •[3] $x = 0, 2$ •[4] when $x = 2$; $y = 2 + (2)^2 - \frac{1}{3}(2)^3 = 3\frac{1}{3}$ •[5] $A(2,\ 3\frac{1}{3})$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4(a)	**ans: proof; 2 : 1** **(3 marks)**	
	•[1] finds $\overrightarrow{PR}$ and $\overrightarrow{RQ}$	•[1] $\overrightarrow{PR} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$; $\overrightarrow{RQ} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$
	•[2] conclusion	•[2] since $\overrightarrow{PR} = 2\overrightarrow{RQ}$, PR is parallel to RQ but as R is a common point P, Q and R are collinear.
	•[3] states ratio	•[3] 2 : 1
(b)	**ans: proof** **(4 marks)**	
	•[1] finds $\overrightarrow{PS}$ and $\overrightarrow{PR}$	•[1] $\overrightarrow{PS} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$; $\overrightarrow{PR} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$
	•[2] finds $\overrightarrow{PS}.\overrightarrow{PR}$	•[2] $\overrightarrow{PS}.\overrightarrow{PR} = 12 - 6 + 12 = 18$
	•[3] finds magnitudes of $\overrightarrow{PS}$ and $\overrightarrow{PR}$	•[3] $\lvert\overrightarrow{PS}\rvert = \sqrt{27}$; $\lvert\overrightarrow{PR}\rvert = 6$
	•[4] substitutes into formula and simplifies	•[4] $\cos SPR = \frac{18}{6\sqrt{27}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$
5	**ans: $y = 9x - 16$** **(5 marks)**	
	•[1] knows to differentiate	•[1] $\frac{dy}{dx} =$
	•[2] finds derivative	•[2] $3x^2 - 3$
	•[3] substitutes $x = 2$ in derivative	•[3] $3(2)^2 - 3 = 9$
	•[4] finds point on the line	•[4] $y = (2)^3 - 3(2) = 8 - 6 = 2$; $(2, 2)$
	•[5] substitutes in equation	•[5] $y - 2 = 9(x - 2)$
6	**ans: $3(x - 1)^2 + 8$** **(3marks)**	
	•[1] takes common factor	•[1] $3(x^2 - 2x) + 11$
	•[2] completes square in bracket	•[2] $3[(x - 1)^2 - 1] + 11$
	•[3] simplifies	•[3] $3(x - 1)^2 - 3 + 11 = 3(x - 1)^2 + 8$

	Give 1 mark for each	Illustration(s) for awarding each mark
7(a)	**ans: 0·022** **(3 marks)**	
	●1 subs values to find k	●1 $0\cdot 8 = 1\times e^{-10k}; e^{-10k} = 0\cdot 8$
	●2 takes ln of both sides	●2 $\ln e^{-10k} = \ln 0\cdot 8$
	●3 releases power and solves for k	●3 $k = \frac{\ln 0\cdot 8}{-10} = 0\cdot 022$
(b)	**ans: 31·5 days** **(3 marks)**	
	●1 re-writes formula and takes ln of both sides	●1 $0\cdot 5 = e^{-0\cdot 022t}; \ln e^{-0\cdot 022t} = \ln 0\cdot 5$
	●2 finds expression for t	●2 $t = \frac{\ln 0\cdot 5}{-0\cdot 022}$
	●3 solves for t	●3 $t = 31\cdot 5$ days
8	**ans:** $f(x) = 2\cos 4x - 1$ **(4 marks)**	
	●1 knows to find integral	●1 $\int -8\sin 4x\ dx$
	●2 finds integral including adding C	●2 $2\cos 4x + C$
	●3 knows to substitute given values	●3 $2\cos(\frac{2\pi}{3}) + C = -2$
	●4 finds value of C and states $f(x)$	●4 $2\times(-\frac{1}{2}) + C = -2; C = -1;$ $f(x) = 2\cos 4x - 1$
9(a)	**ans:** Yes , $104\cdot 08 > 100$ **(3 marks)**	
	●1 taking $a = 0\cdot 97$	●1 $a = 0\cdot 97$
	●2 calculation	●2 $U_{12} = (0\cdot 97)^{12}\times 150 = 104\cdot 08$
	●3 consistent answer	●3 Yes , since $U_{12} > 100$
(b)	**ans:** Plan o.k., over the long-term between $113\cdot 3$ *and* $163\cdot 3$ mgs **(4 marks)**	
	●1 attempting lines of working	●1 $U_1 \to U_5$ below ……… 154·08, 156·91, 158·87, 160·23, 161·17
	●2 finding the limit	●2 $L = \frac{50}{1-(0\cdot 97)^{12}} = 163\cdot 31$
	●3 being aware of the lower limit as well as the upper limit	●3 lower limit = $163\cdot 31 - 50 = 113\cdot 31$
	●4 Consistent comment on findings	●4 Over the long-term the amount present would always be between 113·31 and 163·31 which is ideal.

	Give 1 mark for each •	Illustration(s) for awarding each mark
10	**ans: 2·67** **(5 marks)**	
	•1 prepares to differentiate	•1 $A(r) = 120r^{-1} + \pi r^2$
	•2 differentiates	•2 $A'(r) = -120r^{-2} + 2\pi r$
	•3 equates derivative to 0	•3 $-120r^{-2} + 2\pi r = 0$
	•4 solves for r	•4 $r = 2{\cdot}67$
	•5 justifies answer	•5 $\searrow \rightarrow \nearrow$ or other acceptable method
11	**ans:** $a = \frac{2}{3}$ **(7 marks)**	
	•1 setting up integral	•1 $A = \int_1^{1+a} (6x-2)\ dx$
	•2 integrating correctly	•2 $= \left[\ 3x^2 - 2x\ \right]_1^{1+a}$
	•3 making integral equal 4	•3 $\left[\ 3x^2 - 2x\ \right]_1^{1+a} = 4$
	•4 substituting	•4 $(3(1+a)^2 - 2(1+a)) - (1) = 4$
	•5 simplifying to quadratic equ.	•5 $3a^2 + 4a - 4 = 0$
	•6 factorising	•6 $(3a-2)(a+2) = 0$
	•7 solving to answer	•7 $\therefore\ a = \frac{2}{3}$ (note: -2 is a discard)
12(a)	**ans:** $4y = 3x - 36$ (or equiv.) **(4 marks)**	
	•1 extracting centres coordinates	•1 C(–2,2)
	•2 gradient of radius	•2 $m_r = \frac{-6-2}{4+2} = -\frac{4}{3}$
	•3 gradient of tangent	•3 $m_{\tan} = \frac{3}{4}$
	•4 substitution to answer	•4 $y + 6 = \frac{3}{4}(x-4)$
(b)	**ans:** B(12,0) **(2 marks)**	
	•1 knowing $y = 0$	•1 $4(0) = 3x - 36\ \therefore\ x = 12$
	•2 stating point	•2 B(12, 0)
(c)	**ans:** $(x-5)^2 + (y-1)^2 = 50$ **(3 marks)**	
	•1 locating centre	•1 mid-pt. of BC .... M(5, 1)
	•2 for calculating r^2	•2 $r^2 = 7^2 + 1^2 = 50$
	•3 substituting in equ. to answer	•3 $(x-5)^2 + (y-1)^2 = 50$
		Total: 70 marks