H

National Qualifications
CFE Higher Mathematics - Specimen Paper D

Mathematics
Paper 1
(Non-Calculator)

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

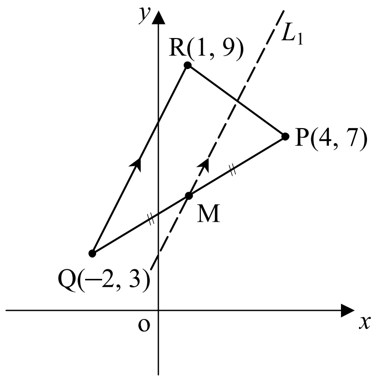
$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

Attempt ALL questions

Total marks — 60

1. Triangle PQR has vertices P(4, 7) , Q(–2, 3) and R(1, 9) as shown.

M is the mid-point of QP.



(a) Find the equation of the line, L_1, which passes through M and is parallel to QR. **3**

(b) Hence verify that L_1 also passes through the mid-point of RP. **2**

2. Given that $f(x) = \left(x + \dfrac{1}{x}\right)^2$, $x > 0$, evaluate $f'(1)$. **4**

3. Solve algebraically the equation

$$\sqrt{2}\sin^2 x = \sin x \quad , \qquad 0 \le x < 2\pi .$$ **4**

4. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n - 2$, where a is a constant.

(a) Given that $U_0 = 20$, show that, in terms of a, $U_2 = 20a^2 - 2a - 2$. **2**

(b) Hence find a, where $a > 0$, given that $U_2 = 2$. **2**

(c) Establish the limit of this sequence as $n \to \infty$. **2**

5. An equation is given as $\frac{5(k-2)}{x} = x + 2(2-k)$, where $x \neq 0$.

(a) Show clearly that this equation can be written in the form

$$x^2 + (4-2k)x + (10-5k) = 0 .$$ **2**

(b) Hence find the values of k which would result in the above equation having **equal roots**. **4**

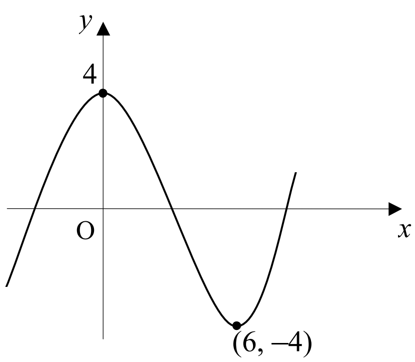
6. Find $\int \frac{x+1}{\sqrt{x}}\, dx$. **4**

7. A function is given as $f(x) = \cos^2\theta - \cos\theta - 1$ for $0 \leq \theta \leq \pi$.

Express the function in the form $f(x) = (\cos\theta + p)^2 + q$. **3**

8. Given that $\log_3(x+5) - \log_3 2 = \log_2 4$, find the value of x . **4**

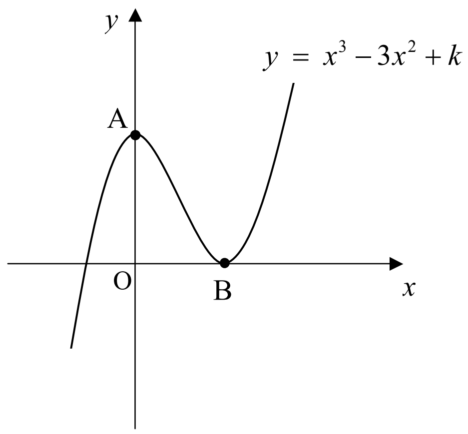
9. The diagram below shows part of the graph of $y = g(x)$.
The function has stationary points at (0,4) and (6, –4).



Sketch the graph of the derived function $y = g'(x)$. **3**

10. Part of the curve with equation $y = x^3 - 3x^2 + k$, where k is a constant, is shown below.

The curve has stationary points on the axes at A and B.



(a) Find the coordinates of the stationary point B. **3**

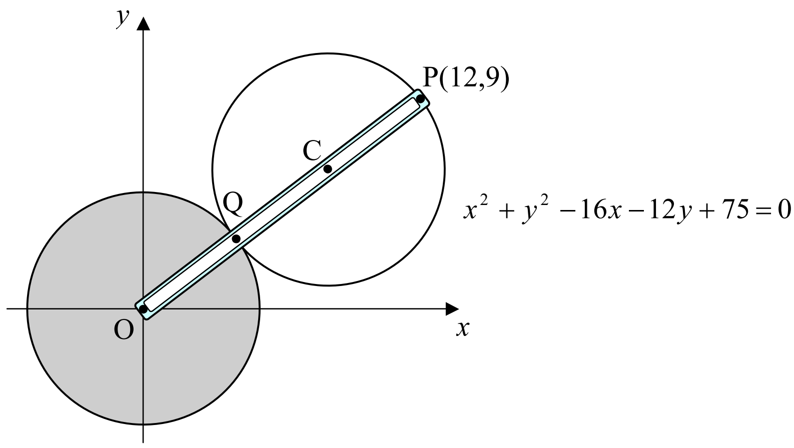
(b) Hence, establish the value of k, and write down the coordinates of the other stationary point A. **3**

11. The diagram below shows two circles locked together by a connecting rod OP.

The circle, centre C, has as its equation $x^2 + y^2 - 16x - 12y + 75 = 0$ and has PQ as a diameter.

P is the point (12,9) as shown.

The shaded circle is centred on the origin and has OQ as a radius.



(a) Write down the coordinates of C. **1**

(b) Hence find the coordinates of Q. **2**

(c) Establish the equation of the shaded circle. **2**

12. **(a)** Show that $(x - 2)$ is a factor of $3x^3 - 8x^2 + 3x + 2$ **3**

(b) Hence, solve $3x^3 - 8x^2 + 3x + 2 = 0$ **3**

13. For what value(s) of x is the function

$\frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x$ increasing ? **4**

[**END OF QUESTION PAPER**]

H

National Qualifications
CFE Higher Mathematics - Specimen Paper D

Mathematics Paper 2 (Calculator)

Duration — 1 hour and 30 minutes

Total marks — 70

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2+y^2+2gx+2fy+c=0$ represents a circle centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$.

The equation $(x-a)^2+(y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

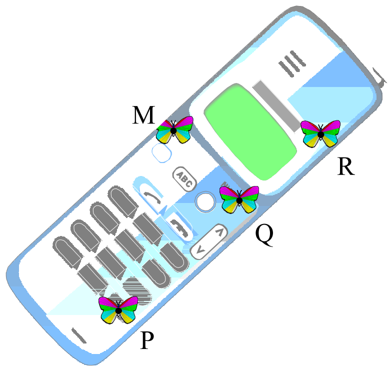
Attempt ALL questions

Total marks — 70

1. Show that the line with equation $3x - y - 10 = 0$ is a tangent to the circle $x^2 + y^2 = 10$ and find the point of contact. **5**

2. Find the gradient of the tangent to the curve with equation $y = 2\cos 3x - \sin^2 x$ at the point with x-coordinate $\frac{\pi}{2}$. **4**

3. Relative to an origin, three butterflies P, Q and R, on a toy mobile phone, have coordinates P(0, –3, –1), Q(4, –5, 3) and R(6, –6, 5).



(a) Show that these three butterflies all lie on the same straight line and write down the ratio in which Q divides PR. **4**

(b) Another butterfly, M, is added at the point (a, –5, 1) making QM perpendicular to PR.

Calculate the value of a. **5**

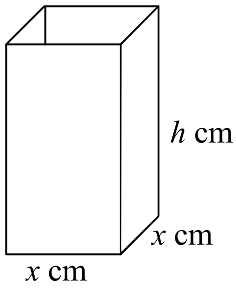
(c) Calculate the size of the angle PMR. **5**

4, **(a)** Express $\sqrt{7}\cos x^\circ + 3\sin x^\circ$ in the form $k\sin(x - \alpha)^\circ$. **5**

(b) Hence, write down the minimum value of $\dfrac{14}{3 + \sqrt{7}\cos x^\circ + 3\sin x^\circ}$. **2**

5. A curve for which $\dfrac{dy}{dx} = 4x + 1$ passes through the point $(1, -1)$. Express y in terms of x. **4**

6. The diagram shows an open top box with a square base of x cm and height h cm.



The box has to be made from $1350\,\text{cm}^2$ of card.

(a) Show that, in terms of x, the height, $h$cm, of the box can be expressed as

$$\frac{1350 - x^2}{4x}$$ **2**

(b) Show clearly that the volume of the box, in terms of x, can be expressed as:

$$V(x) = \tfrac{1}{4}x(1350 - x^2)$$ **3**

(c) Hence, or otherwise, find the value of x, so that the volume is a maximum, leaving your answer as a surd in its simplest form. **Justify your answer.** **5**

7. A radioactive substance decays according to the formula $M_t = M_o e^{-0\cdot004t}$ where M_o is the initial mass of the substance and M_t is the mass remaining after t days.

(a) Show that the time taken for the substance to lose half its mass can be written as $t = 250\log_e 2$. **4**

(b) If the initial mass was 500g, calculate the mass remaining after 500 days. **2**

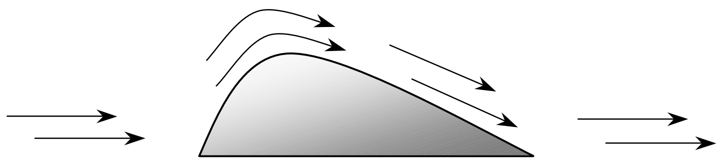
8. Two functions are defined on suitable domains as

$$f(x) = \frac{x-2p}{3} \quad \text{and} \quad g(x) = x^2 + p \quad \text{, where } p \text{ is a constant.}$$

Show clearly that the composite function $g(f(x))$ can be expressed in the form

$$g(f(x)) = \tfrac{1}{9}\left(x^2 - 4px + 4p^2 + 9p\right)$$ **4**

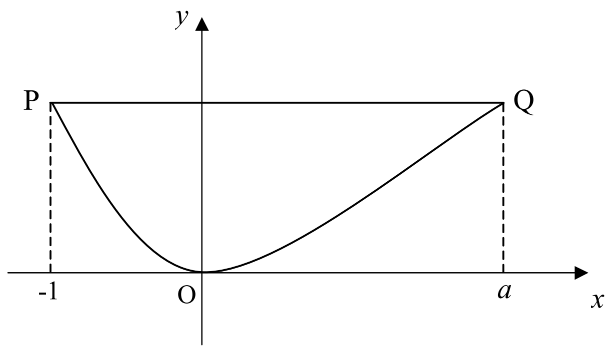
9. A designer is working on a new competition helmet for olympic ski racers. He has perfected the design to produce as little drag and as even a wind-flow over the helmet as possible.



Below is part of his computer aided design showing a flat cross-section of the helmet relative to a set of rectangular axis. The helmet has been rotated through 180°.

The curve PQ has as its equation $y = 3x^2 - x^3$. The line PQ is horizontal.

The x-coordinates of P and Q are -1 and a respectively.



(a) **Show clearly** that the equation of the line PQ is $y = 4$. **1**

(b) Hence determine the value of a. **3**

(c) Calculate the **area** enclosed between the line PQ and the curve with equation $y = 3x^2 - x^3$.
Give your answer in square units. **4**

10. **(a)** Find the exact values of $\sin\theta$ and $\cos\theta$. **3**

(b) Hence show clearly that the exact value of $\sin(\theta + \frac{\pi}{3})$ can be expressed as

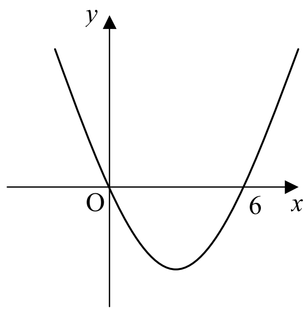
$$\sin(\theta + \tfrac{\pi}{3}) = \tfrac{1}{6}(\sqrt{6} + 3).$$ **5**

[**END OF QUESTION PAPER**]

CFE Higher Specimen Paper D – Marking Scheme Paper 1

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: $y = 2x + 3$** **(3 marks)**	
	•[1] finding the coords. of M	•[1] M(1,5
	•[2] obtaining correct gradient	•[2] $m = \frac{9-3}{1-(-2)} = \frac{6}{3} = 2$
	•[3] finding equation of line	•[3] $y - 5 = 2(x-1)$
(b)	**ans: proof** **(2 marks)**	
	•[1] strategy of mid-point into line	•[1] N($2\frac{1}{2}$, 8) ..... into $y = 2x + 3$
	•[2] correct substitution to answer	•[2] $8 = 2(2\frac{1}{2}) + 3$
2	**ans: 0** **(4 marks)**	
	•[1] expanding brackets	•[1] $x^2 + 2 + \frac{1}{x^2}$
	•[2] preparing for differentiation	•[2] $x^2 + 2 + x^{-2}$
	•[3] differentiating	•[3] $2x - 2x^{-3}$
	•[4] substituting to answer	•[4] $f'(1) = 2(1) - \frac{2}{1^3} = 0$
3	**ans:** $\left\{0 , \frac{\pi}{4} , \frac{3\pi}{4} , \pi\right\}$ **(4 marks)**	
	•[1] re-arranging to zero	•[1] $\sqrt{2}\sin^2 x - \sin x = 0$
	•[2] factorising and solving	•[2] $\sin x(\sqrt{2}\sin x - 1) = 0$ $\sin x = 0$, $\sin x = \frac{1}{\sqrt{2}}$
	•[3] C.Factor answers	•[3] 0 , π
	•[4] other factor answers	•[4] $\frac{\pi}{4}$, $\frac{3\pi}{4}$
4(a)	**ans: proof** **(2 marks)**	
	•[1] setting out U_1 & U_2	•[1] $U_1 = 20a - 2$, $U_2 = aU_1 - 2$
	•[2] substit. to answer	•[2] $U_2 = a(20a - 2) - 2 = 20a^2 - 2a - 2$
(b)	**ans:** $a = \frac{1}{2}$ **(2 marks)**	
	•[1] combining and "=0"	•[1] $20a^2 - 2a - 4 = 0$ $2(10a^2 - a - 2) = 0$
	•[2] solving to answer	•[2] $2(5a+2)(2a-1) = 0$ $a = -\frac{2}{5}$ *or* $a = \frac{1}{2}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4(c)	**ans: -4** **(2 marks)**	
	•[1] knowing how to find limit	•[1] $L = \dfrac{b}{1-a}$ .... or equivalent
	•[2] correct answer	•[2] -4
5(a)	**ans: proof** **(2 marks)**	
	•[1] multiplying through by x	•[1] $5(k-2) = x^2 + 2x(2-k)$
	•[2] remaining algebra to answer	•[2] $x^2 + (4-2k)x - (5k-10)$ …….
(b)	**ans: $k = -3$, $k = 2$** **(4 marks)**	
	•[1] statement on equal roots	•[1] $b^2 - 4ac = 0$, stated or implied
	•[2] a,b and c	•[2] $a = 1$, $b = (4-2k)$ & $c = (10-5k)$
	•[3] substituting	•[3] $(4-2k)^2 - 4(10-5k) = 0$ (or equiv)
	•[4] solving to answer	•[4] $4k^2 + 4k - 24 = 0$ $4(k^2 + k - 6) = 0$ $k = -3$, $k = 2$
6	**ans: $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$** **(4 marks)**	
	•[1] dealing with denominator	•[1] $x^{-\frac{1}{2}}(x+1)$ .... or equiv.
	•[2] preparing for integration	•[2] $x^{\frac{1}{2}} + x^{-\frac{1}{2}}$
	•[3] integrating first term	•[3] $\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}$ .... or equiv.
	•[4] integrating second term + C	•[4] $\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$ .... or equiv.
7	**ans: $(\cos\theta - \frac{1}{2})^2 - \frac{5}{4}$** **(3 marks)**	
	•[1] inside bracket	•[1] $(\cos\theta - \frac{1}{2})^2$
	•[2] subtracting a quarter	•[2] $(\cos\theta - \frac{1}{2})^2 - \frac{1}{4}$
	•[3] tidying to answer	•[3] $(\cos\theta - \frac{1}{2})^2 - \frac{5}{4}$

	Give 1 mark for each	Illustration(s) for awarding each mark
8	**ans:** $x = 13$ **(4 marks)**	
	●1 combining L.H. logs	●1 $\log_3 \frac{x+5}{2} = \log_2 4$
	●2 evaluating R.H. log	●2 .... $= 2$
	●3 removing logs.	●3 $3^2 = \frac{x+5}{2}$
	●4 calculating ans.	●4 $x = 13$
9	**ans:** see sketch opposite **(3 marks)**	
	●1 relating stat. points x-coordinates onto x-axis	●1
	●2 parabolic shape	●2
	●3 annotation	●3
10(a)	**ans:** B(2, 0) **(3 marks)**	
	●1 knowing to diff. and solve to zero	●1 $\frac{dy}{dx} = 0$ at a stationary point (stated or implied)
	●2 differentiating	●2 $\frac{dy}{dx} = 3x^2 - 6x$ (k goes)
	●3 solving correctly and choosing 2	●3 $3x(x-2) = 0 \ \therefore \ x = 0 \ or \ x = 2$
(b)	**ans:** $k = 4$ and A(0, 4) **(3 marks)**	
	●1 knowing to substitute B in equ. (stated or implied)	●1 (2,0) lies on line … satisfy equation
	●2 solving out to k	●2 $2^3 - 3(2^2) + k = 0$, $\therefore$ $k = 4$
	●3 realising answer for A	●3 for A when $x = 0$ then $y = 4$, A(0 ,4)
11(a)	**ans:** C(8, 6) **(1 mark)**	
	●1 answer	●1 C(8, 6)
(b)	**ans:** Q(4, 3) **(2 marks)**	
	●1 method	●1 any acceptable method
	●2 answer	●2 Q(4, 3)
(c)	**ans:** $x^2 + y^2 = 25$ **(2 marks)**	
	●1 finding r^2 or r by pyth.	●1 $r^2 = 4^2 + 3^2 = 25$
	●2 equation (centred on O)	●2 centred on the origin $\therefore$ $x^2 + y^2 = 25$

	Give 1 mark for each	Illustration(s) for awarding each mark
12(a)	**ans: –1** **(3 marks)**	
	●[1] knows to use $x = 2$	●[1] $2 \,\vert\,$
	●[2] completes synthetic division	●[2] $\begin{array}{r\|rrrr} 2 & 3 & -8 & 3 & 2 \\ & & 6 & -4 & -2 \\ \hline & 3 & -2 & -1 & 0 \end{array}$
	●[3] recognition of zero remainder	●[3] $(x - 2)$ is a factor as remainder is zero
(b)	**ans: $x = 2, 1, -⅓$** **(3 marks)**	
	●[1] identify quotient ●[2] factorises fully ●[3] solves	●[1] $(x - 2)(3x^2 - 2x - 1) = 0$ ●[2] $(x - 2)(3x + 1)(x - 1) = 0$ ●[3] $x = 2, 1, -⅓$
13	**ans: $x < -4$ or $x > 3$** **(4 marks)**	
	●[1] knows to differentiate ●[2] knows derivative > 0 ●[3] attempts to solve for x ●[4] answer	●[1] $x^3 + x - 12$ ●[2] $x^3 + x - 12 > 0$ ●[3] graph drawn (or any acceptable method) ●[4] $x < -4$ or $x > 3$
		Total: 60 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1	**ans: (3, –1)** **(5 marks)**	
	•[1] rearranges to $y = mx + c$	•[1] $y = 3x - 10$
	•[2] substitute in circle equation	•[2] $x^2 + (3x - 10)^2 = 10$
	•[3] multiplies brackets and rearranges	•[3] $10(x^2 - 6x + 9) = 0$
	•[4] proves tangency	•[4] $10(x-3)^2 = 0$ ..one root / $b^2 - 4ac = 0$
	•[5] finds point of contact	•[5] (3, -1)
2	**ans: $m = 6$** **(4 marks)**	
	•[1] knows to take derivative	•[1] $\frac{dy}{dx} = ...$
	•[2] finds derivative	•[2] $= -6\sin 3x - 2\sin x \cos x$
	•[3] substitutes	•[3] $6\sin(\frac{3\pi}{2}) - 2\sin\frac{\pi}{2}\cos\frac{\pi}{2}$
	•[4] evaluates to answer	•[4] $6 - 0 = 6$
3(a)	**ans: proof** **(4 marks)**	
	•[1] preparing displacement $\vec{PQ}$	•[1] $\vec{PQ} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$
	•[2] preparing displacement $\vec{QR}$	•[2] $\vec{QR} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$
	•[3] reason for collinearity and common pt.	•[3] $\vec{PQ} = 2\vec{QR}$, Q is common point
	•[4] ratio	•[4] 2 : 1
(b)	**ans: $a = 6$** **5 marks**	
	•[1] preparing displacement $\vec{QM}$	•[1] $\vec{QM} = \begin{pmatrix} a-4 \\ 0 \\ -2 \end{pmatrix}$
	•[2] preparing displacement $\vec{PR}$	•[2] $\vec{PR} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
	•[3] stating condition for perp. vectors •[4] finding scalar product •[5] solution	•[3] $\vec{QM} \cdot \vec{PR} = 0$ for perp. vectors •[4] $6a - 36 = 0$ •[5] $a = 6$
(c)	**ans: $111{\cdot}4^o$ 5 marks**	
	•[1] construct appropriate vectors •[2] strategy of $\cos\theta = ...$ •[3] calculate scalar product •[4] find magnitudes •[5] calculate angle	•[1] $\vec{MP} = \begin{pmatrix} -6 \\ 2 \\ -2 \end{pmatrix}$, $\vec{MR} = \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$ •[2] $\cos\theta = ...$ (formula may only appear when numbers are substituted) •[3] $\vec{MP} \cdot \vec{MR} = 0 - 2 - 8 = -10$ •[4] $\lvert MP\rvert = \sqrt{44}$, $\lvert MR\rvert = \sqrt{17}$ •[5] $\cos\theta = \dfrac{-10}{\sqrt{44} \times \sqrt{17}} \quad \therefore \quad \theta = 111 \cdot 4^\circ$
4(a)	**ans: $4\sin(x - 318 \cdot 6)^o$ (5 marks)**	
	•[1] correct expansion •[2] equating coefficients •[3] tan ratio + correct quadrant •[4] value of k •[5] value of α	•[1] $= k\sin x^\circ \cos\alpha^\circ - k\cos x^\circ \sin\alpha^\circ$ •[2] $k\cos\alpha^\circ = 3$, $k\sin\alpha^\circ = -\sqrt{7}$ •[3] $\tan\alpha^\circ = \dfrac{-\sqrt{7}}{3}$, 4th quadrant •[4] $k = \sqrt{16} = 4$ •[5] $\alpha = 318 \cdot 6^o$
(b)	**ans: 2 (2 marks)**	
	•1 using max. value from above •[2] answer	•[1] $\dfrac{14}{3+4}$ •[2] 2
5	**ans: $y = 2x^2 + x - 4$ (4 marks)**	
	•[1] knows to integrate •[2] integrates and includes c •[3] substitutes •[4] finds c and express y in terms of x	•[1] evidence of setting up integral •[2] $y = 2x^2 + x + c$ •[3] $-1 = 2 + 1 + c$ •[4] $y = 2x^2 + x - 4$

	Give 1 mark for each	Illustration(s) for awarding each mark
6(a)	**ans: proof** **(2 marks)**	
	•1 finds expression for S.A.	•1 $x^2 + 4xh = 1350$
	•2 rearranges to answer	•2 $4xh = 1350 - x^2$; $h = \dfrac{1350 - x^2}{4x}$
(b)	**ans: proof** **(3 marks)**	
	•1 finds expression for volume	•1 $V = x \times x \times \left(\dfrac{1350 - x^2}{4x}\right)$
	•2 cancels	•2 $V = x \times \left(\dfrac{1350 - x^2}{4}\right)$
	•3 completes rearranging	•3 $V = \dfrac{1}{4}x\left(1350 - x^2\right)$
(c)	**ans: 15√2** **(5 marks)**	
	•1 knows to differentiate and equal 0	•1 $\dfrac{dV}{dx} = 0$
	•2 differentiates	•2 $\dfrac{1350}{4} - \dfrac{3}{4}x^2$
	•3 solves for x	•3 $x = \sqrt{450}$
	•4 expresses as a surd	•4 $15\sqrt{2}$
	•5 justifies answer	•5 $\nearrow \rightarrow \searrow$ or other acceptable method
7(a)	**ans: proof** **(4 marks)**	
	•1 substitutes value for m_t and uses $\frac{1}{2} = 2^{-1}$	•1 $\frac{1}{2} = e^{-0\cdot004t}$; $2^{-1} = e^{-0\cdot004t}$
	•2 takes $\log_e$ of both sides	•2 $-\log_e 2 = \log_e e^{-0\cdot004t}$
	•3 simplifies	•3 $-\log_e 2 = -0\cdot004t$
	•4 realises $0\cdot004 = \frac{1}{250}$ and rearranges to ans	•4 $\log_e 2 = \frac{1}{250}t; \rightarrow 250\log_e 2 = t$
(b)	**ans: 67·7g** **(2 marks)**	
	•1 substitutes values	•1 $500 \times e^{-0\cdot004\times500}$
	•2 evaluates to answer	•2 67·7g

	Give 1 mark for each •	Illustration(s) for awarding each mark
8	**ans:** proof **(4 marks)**	
	•¹ set up composite function	•¹ $g(f(x)) = \left(\dfrac{x-2p}{3}\right)^2 + p$
	•² squaring out bracket	•²&³ $g(f(x)) = \dfrac{x^2-4px+4p^2}{9} + p$
	•³ for the 9	
	•⁴ desired form	•⁴ $g(f(x)) = \frac{1}{9}\left(x^2-4px+4p^2+9p\right)$
9(a)	**ans: proof (1 mark)**	
	•¹ clear working to answer	•¹ $y = 3(-1^2)-(-1^3) = 3-(-1) = 4$ horizontal line $\therefore\ y = 4$
(b)	**ans:** $a = 2$ **(3 marks)**	
	•¹ knowing to solve equ. of curve to	•¹ $3x^2 - x^3 = 4$
	•² arranging to zero and synth. division	•² $-x^3+3x^2-4=0$ $-1 \quad 3 \quad 0 \quad -4$
	•³ finding other root	•³ $\begin{array}{r\|rrrr} 2 & -1 & 3 & 0 & -4 \\ & & -2 & 2 & 4 \\ \hline & -1 & 1 & 2 & 0 \end{array}$
(c)	**ans:** $6\frac{3}{4}$ **units²** **(4 marks)**	
	•¹ setting up integral	•¹ $A = \int_{-1}^{2} 4-\left[3x^2-x^3\right]\,dx$
	•² integrating correctly	•² $A = \left[\,4x - x^3 + \frac{x4}{4}\right]_{-1}^{2}$
	•³ substituting limits of integration	•³ $A = (8-8+4)-(-4+1+\frac{1}{4})$
	•⁴ calculating answer	•⁴ $A = (4)-(-2\frac{3}{4}) = 6\frac{3}{4}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
10(a)	**ans:** $\sin\theta = \frac{2}{\sqrt{6}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{6}}$ **(3 marks)**	
	•[1] drawing a R.A. triangle	•[1] drawing triangle
	•[2] calculating hypotenuse	•[2] $h^2 = 2 + 4 = 6 \;\therefore\; h = \sqrt{6}$
	•[3] lifting answers	•[3] $\sin\theta = \frac{2}{\sqrt{6}}$, $\cos\theta = \frac{\sqrt{2}}{\sqrt{6}}$
(b)	**ans:** proof **(5 marks)**	
	•[1] expanding	•[1] $\sin(\theta + \frac{\pi}{3}) = \sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3}$
	•[2] putting in all exact values	•[2] $= \frac{2}{\sqrt{6}}(\frac{1}{2}) + \frac{\sqrt{2}}{\sqrt{6}}(\frac{\sqrt{3}}{2})$
	•[3] simplifying	•[3] $= \frac{1}{\sqrt{6}} + \frac{1}{2}$
	•[4] rationalising the denominator	•[4] $= \frac{\sqrt{6}}{6} + \frac{1}{2}$
	•[5] taking out common factor to answer	•[5] $\sin(\theta + \frac{\pi}{3}) = \frac{1}{6}(\sqrt{6} + 3)$
		Total: 70 marks